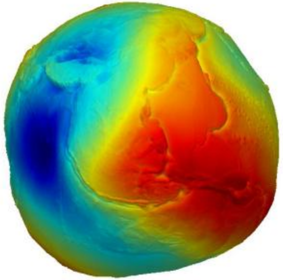
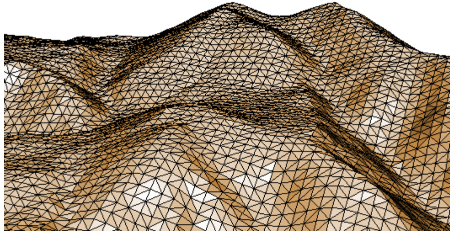
Numerical solution of the fixed gravimetric BVP on the Earth's surface – its possible contribution to the realization of IHRS.





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Contents

- Realization of Vertical Reference Systems
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- Possible advantages for the realization of IHRS

Realization of Vertical Reference Systems

Global approach

• ***globally homogenous approach based on precise gravity field modelling***

$$W_P = U_P(h^{GNSS}) + T_P \qquad c_P = -(W_P - W_0)$$

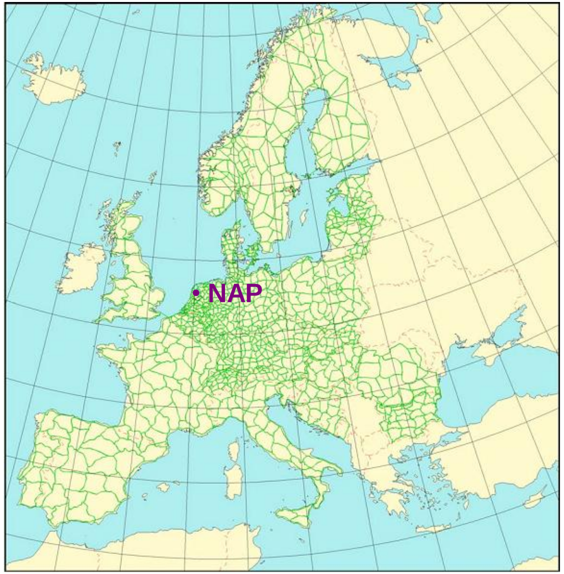
GRACE/GOCE-based satellite-only GGMs

⇒ ***fully independent from LVDs***
⇒ ***low-frequency part obtained very precisely, however overall accuracy affected by the truncation error***

Continental approach (e.g. EVRF2007)

• ***regional approach based on spirit levelling and potential of the height reference surface W_{0i}***

$$c_P = c_{Pi} + W_0 - W_{0i} \Longrightarrow c_{Pi} = W_{0i} - W_P = \int_{0i}^{P} g\,dh$$

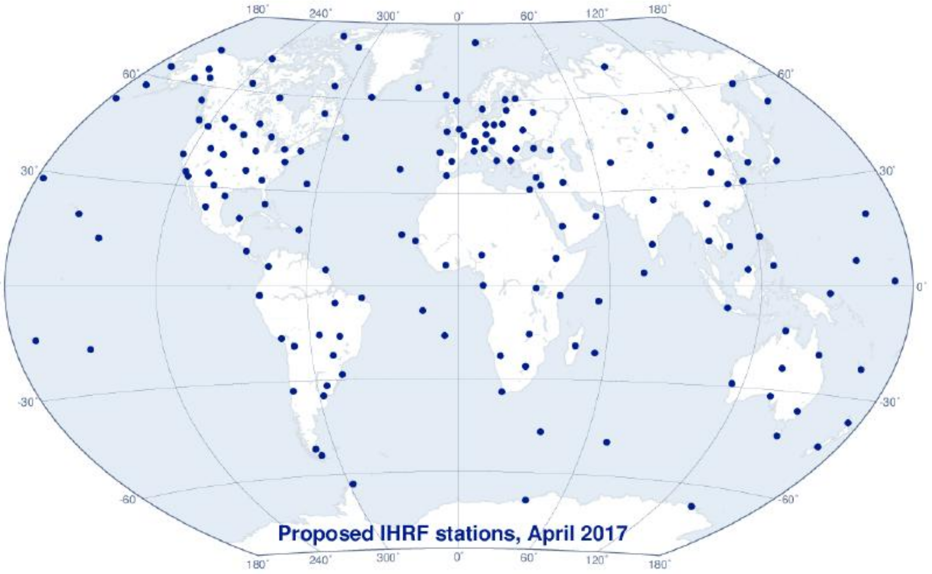


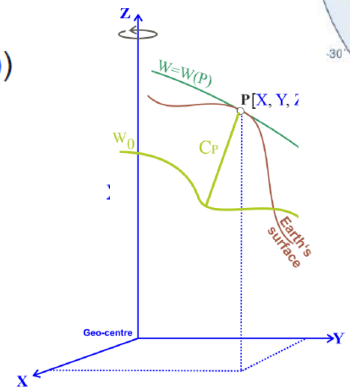
A concept of the realization of IHRS

From presentation of Sánchez at al. 2017 (IAG-IASPEI-2017, KOBE, JAPAN):

1) Vertical coordinates are potential differences with respect to a conventional W_0 value:
   - $C_P = C(P) = W_0 - W(P) = -\Delta W(P)$
   - conventional fixed value

     $W_0 = const. = 62\ 636\ 853.4\ \mathrm{m^2s^{-2}}$
2) The position P is given by the coordinates vector $\mathbf{X}_P$ (X_P, Y_P, Z_P) in the ITRF; i.e., $W(P) = W(\mathbf{X}_P)$
3) The estimation of $\mathbf{X}(P)$, $W(P)$ (or $C(P)$) includes their variation with time; i.e., $\dot{\mathbf{X}}(P)$, $\dot{W}(P)$ (or $\dot{C}(P)$).
4) Coordinates are given in mean-tide system / mean (zero) crust.





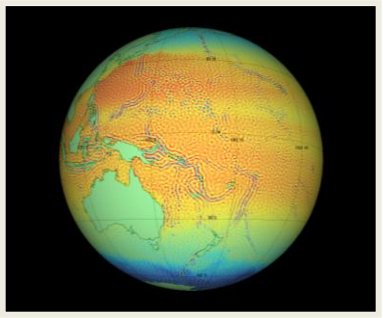
- ***crucial point: to determine W(P) on the Earth's surface***
- ***no need to know 3D position of geoid*** (W_0)

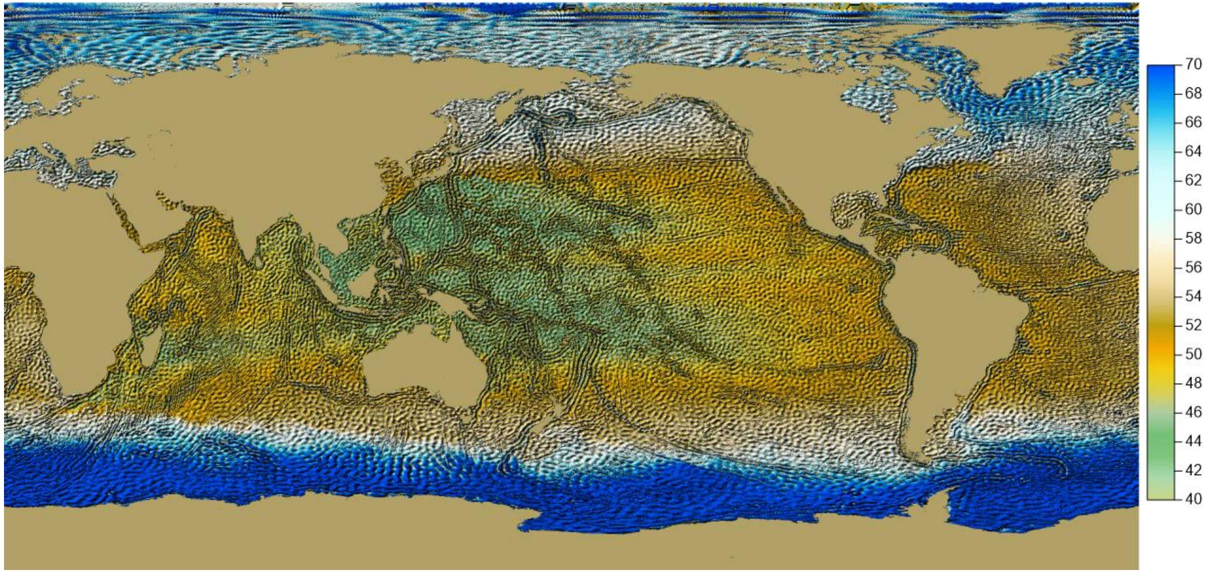
Geopotential at points on the Earth's surface

GOCE-based satellite-only GGMs

⇒ ***low-frequency part obtained very precisely (goal of GOCE) :***
"accuracy of 1 to 2 cm and a spatial resolution of about 100 km"

⇒ ***affected significantly by stripping noise due to omission errors!!!***





• *amplitudes of "dm-level"*
(in some places exceeding 1 m)

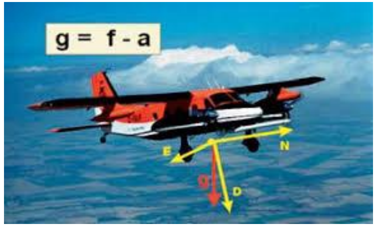
Geopotential on DTU13 mean sea surface evaluated from GO_CONS_GCF_2_DIR_R5
(SH up to d/o 300)

Geopotential at points on the Earth's surface

GOCE-based satellite-only GGMs

⇒ *low-frequency part obtained very precisely (goal of GOCE) :*
"accuracy of 1 to 2 cm and a spatial resolution of about 100 km"

⇒ *affected significantly by stripping noise due to omission errors!!!*



(*Source: Kreye et al. 2006*)

inevitable to model the high-frequency part

- *combined GGMs (including RTM)*
- *national (quasi)geoid models*

!!! *terrestrial or airborne gravimetric measurements* !!!

Fixed gravimetric BVP

Linearized Fixed Gravimetric BVP

– exterior BVP for the Laplace equation

$$\Delta T(\mathbf{x}) = 0 \qquad \mathbf{x} \in ext.\ \Omega$$
$$\langle \nabla T(\mathbf{x}), \mathbf{s}(\mathbf{x}) \rangle = -\,\delta g(\mathbf{x}) \qquad \mathbf{x} \in \Gamma$$
$$T(\mathbf{x}) = O\,(\,|x|^{-1})$$

where $T(\mathbf{x}) = W(\mathbf{x}) - U(\mathbf{x})$ $\quad \Omega$ - *the Earth*

$\mathbf{s}(\mathbf{x}) = -\nabla U(\mathbf{x}) \,/\, |\nabla U(\mathbf{x})|$ $\quad \Gamma$ - *the Earth's surface*

Input data – surface gravity disturbances

(oblique derivative boundary conditions)

$$\delta g(\mathbf{x}) = g(\mathbf{x}) - \gamma(\mathbf{x})$$

Precise 3D positioning by GNSS:

- ***globally consistent***
- ***independent from local vertical datums***

Direct BEM for the fixed gravimetric BVP

Linearized fixed gravimetric BVP

$$\Delta T(\mathbf{x}) = 0 \qquad \mathbf{x} \in ext.\ \Omega$$
$$\langle \nabla T(\mathbf{x}), \mathbf{s}(\mathbf{x}) \rangle = -\delta g(\mathbf{x}) \qquad \mathbf{x} \in \Gamma$$
$$T(\mathbf{x}) = O(|x|^{-1})$$

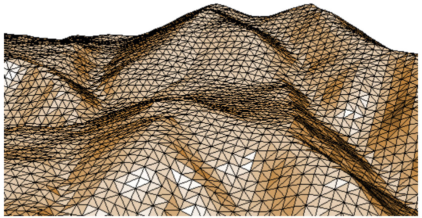
Direct BEM formulation

→ ***Boundary Integral Equation:***

$$\frac{1}{2}T(p) + \int_\Gamma T(q)\frac{\partial G}{\partial n_\Gamma}(p,q)\,d\Gamma_q = \int_\Gamma \frac{\partial T}{\partial n_\Gamma}(q)\,G(p,q)\,d\Gamma_q$$

where

$$G(p,q) = \frac{1}{4\pi \cdot |p-q|}$$

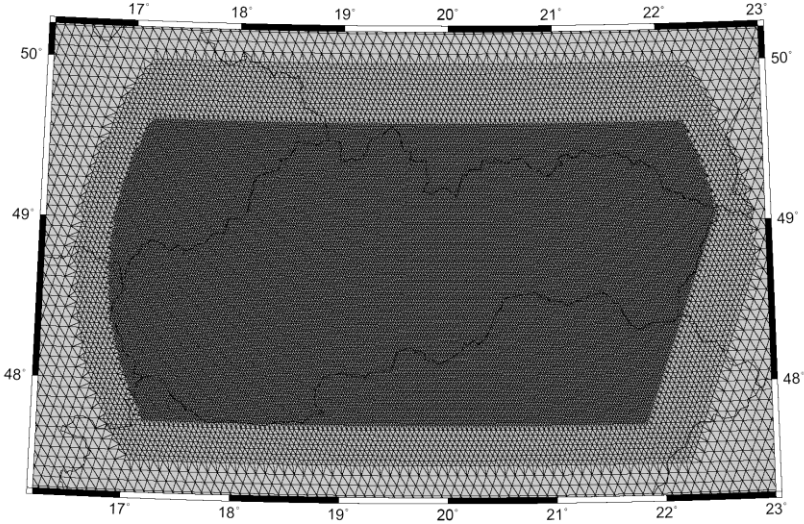


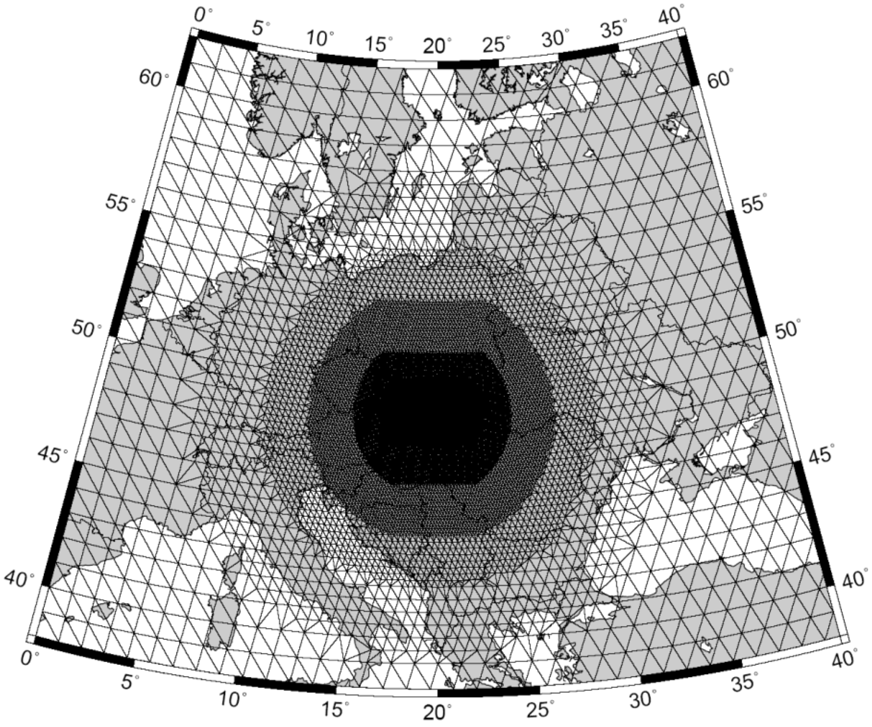
http://www.geom.at/fade2d/html/

$\Rightarrow$ ***fundamental solution of the Laplace equation***
(as a weighted function)

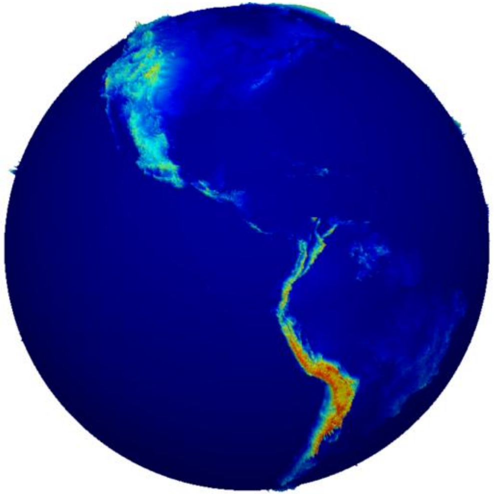
Triangulation of the Earth's surface

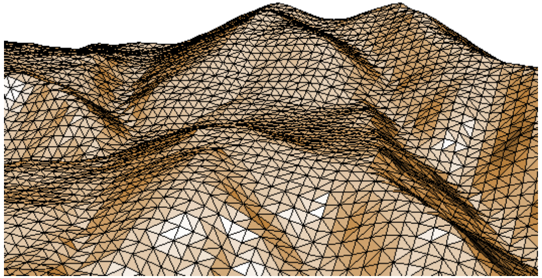
An example of a local refinement of the global triangulation





Triangulation on the Earth's topography





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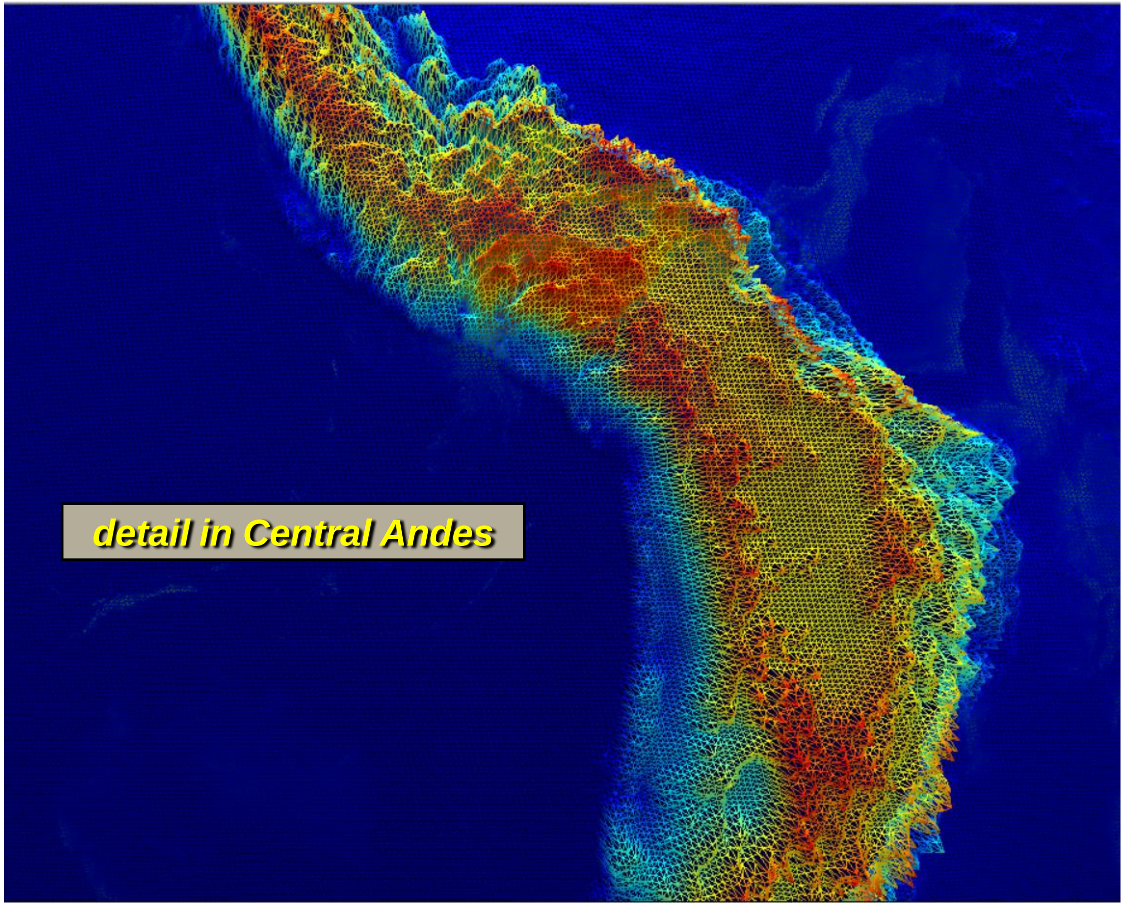


Reconstruction of EGM2008 on the triangulates Earth's topography

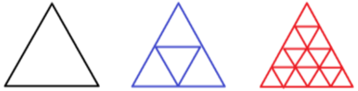
Case A ⇒ **Global resolution:** ***0.075 deg*** ⇒ **5 760 002** *nodes*

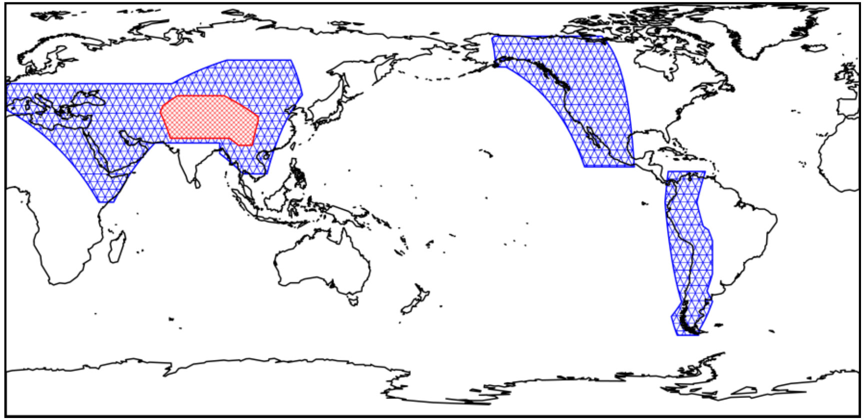
Case B ⇒ **Global resolution:** ***0.05 deg*** ⇒ **12 960 002** nodes

Case C ⇒ **Global resolution:** ***0.075 deg*** ⇒ **8 818 389** nodes

+ local refinement 1: ***0.0375 deg***

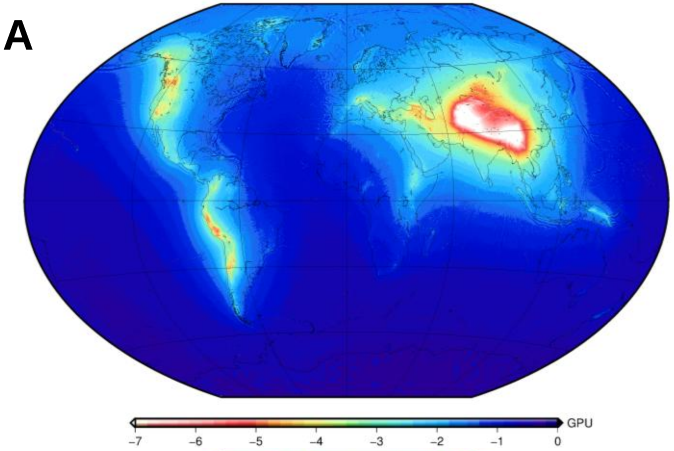
+ local refinement 2: ***0.01875 deg***



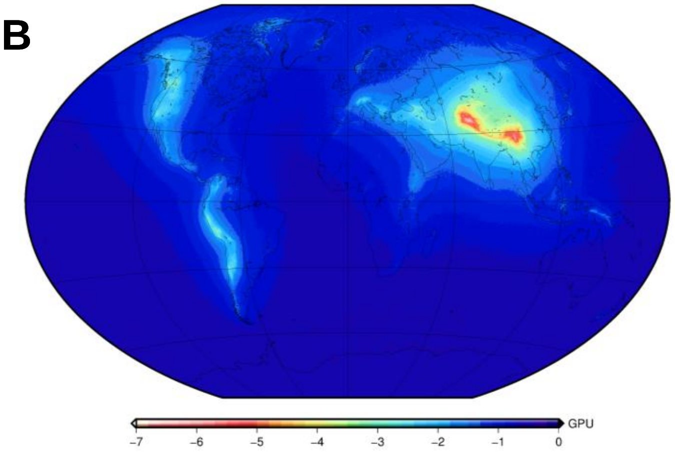


Comparison: BEM - EGM2008

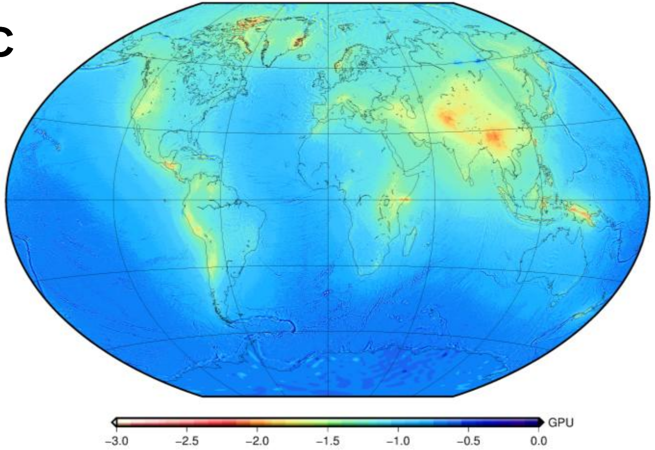
A



B



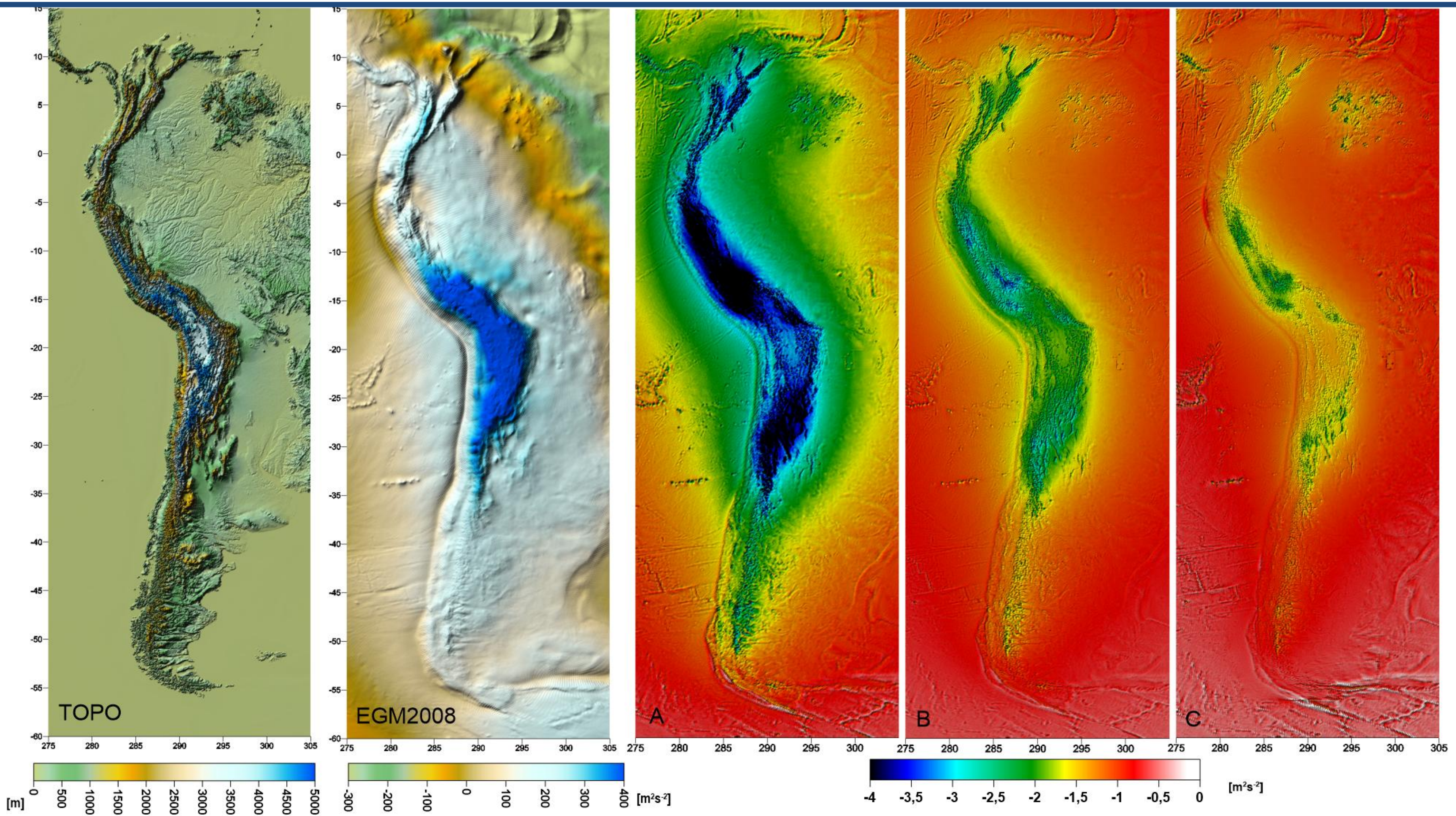
C



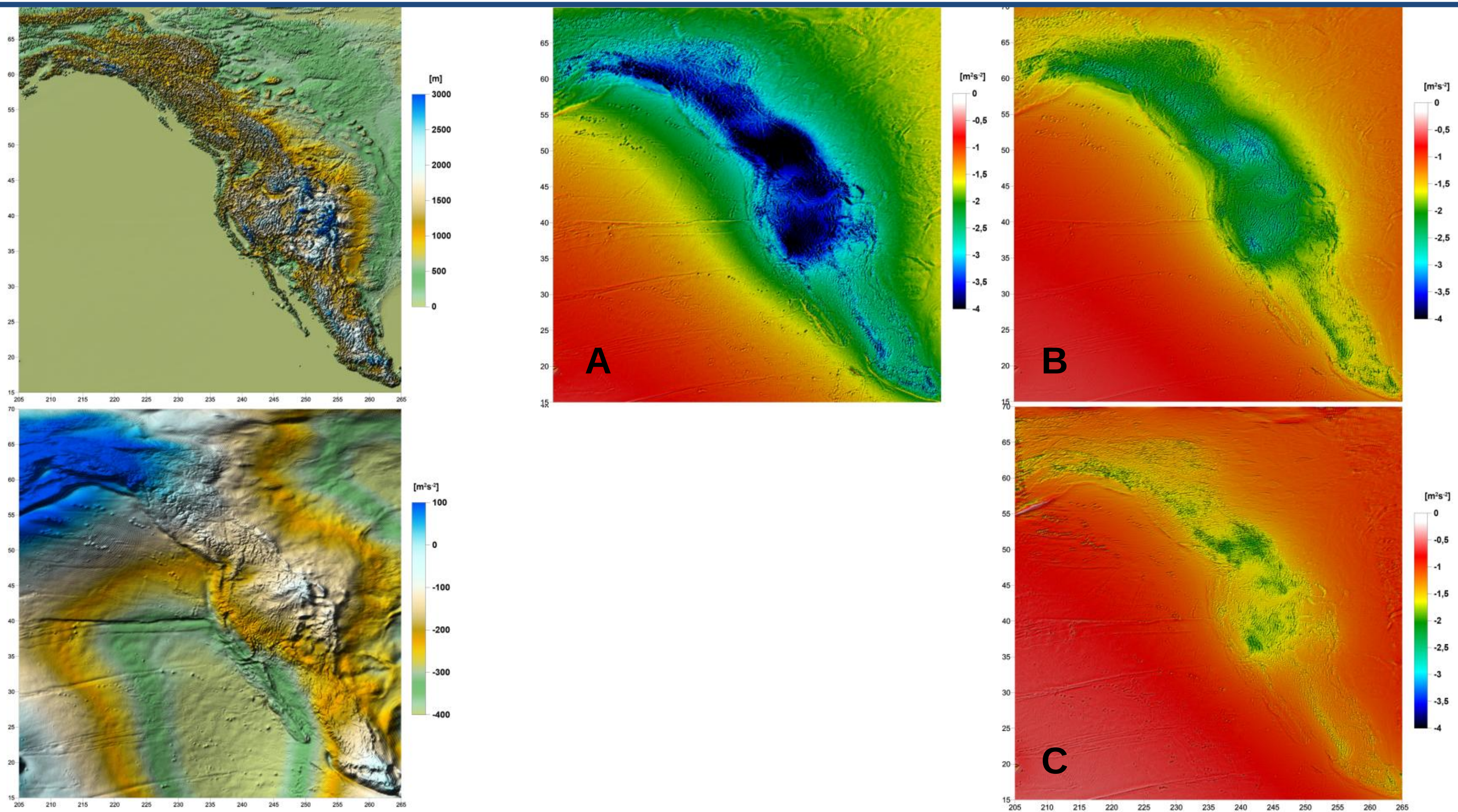
STATISTICS OF RESIDUALS

Case	A	B	C
Resolution	0.075 deg	0.05 deg	A+LR1+LR2
Nodes	5 760 002	12 960 002	8 818 389
Mean [m^2s^{-2}]	-1.315	-0.939	-0.514
Max [m^2s^{-2}]	1.216	0.084	0.663
Min [m^2s^{-2}]	-13.145	-7.320	-4.331
STD [m^2s^{-2}]	1.033	0.564	0.344

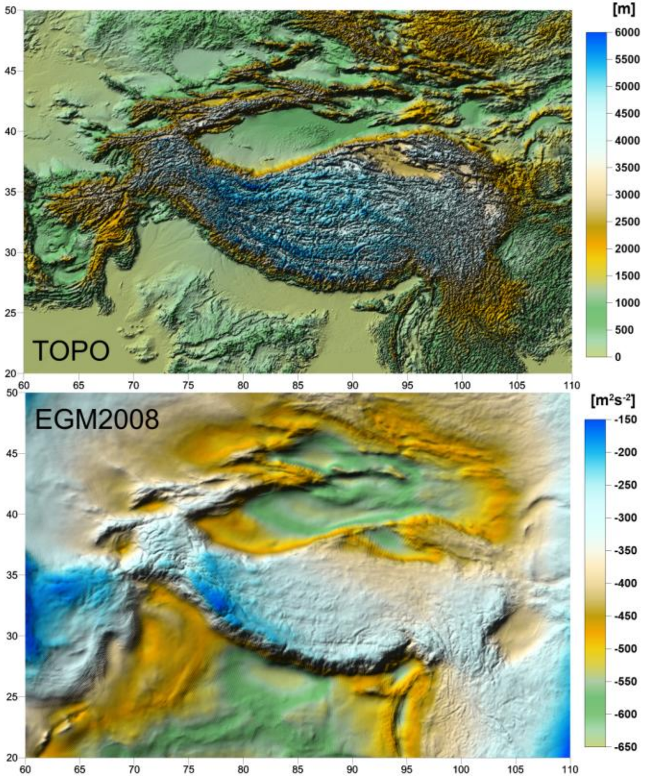
Comparison in Andes: BEM - EGM2008

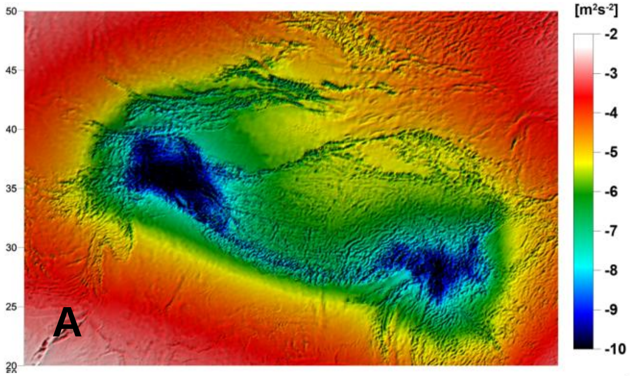


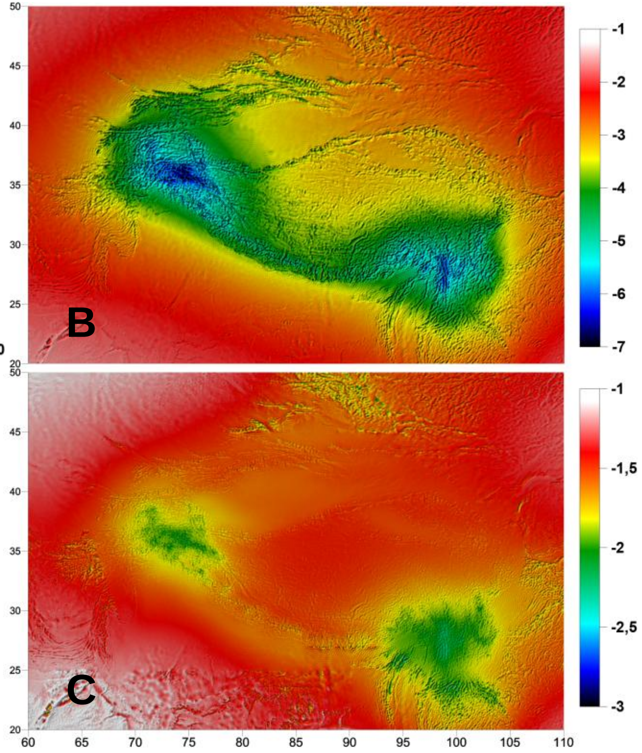
Comparison in North America: BEM - EGM2008

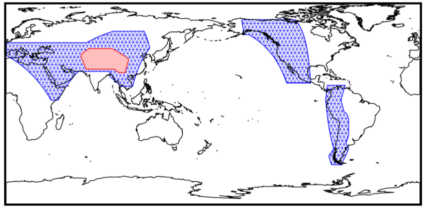


Comparison in Himalayas: BEM - EGM2008

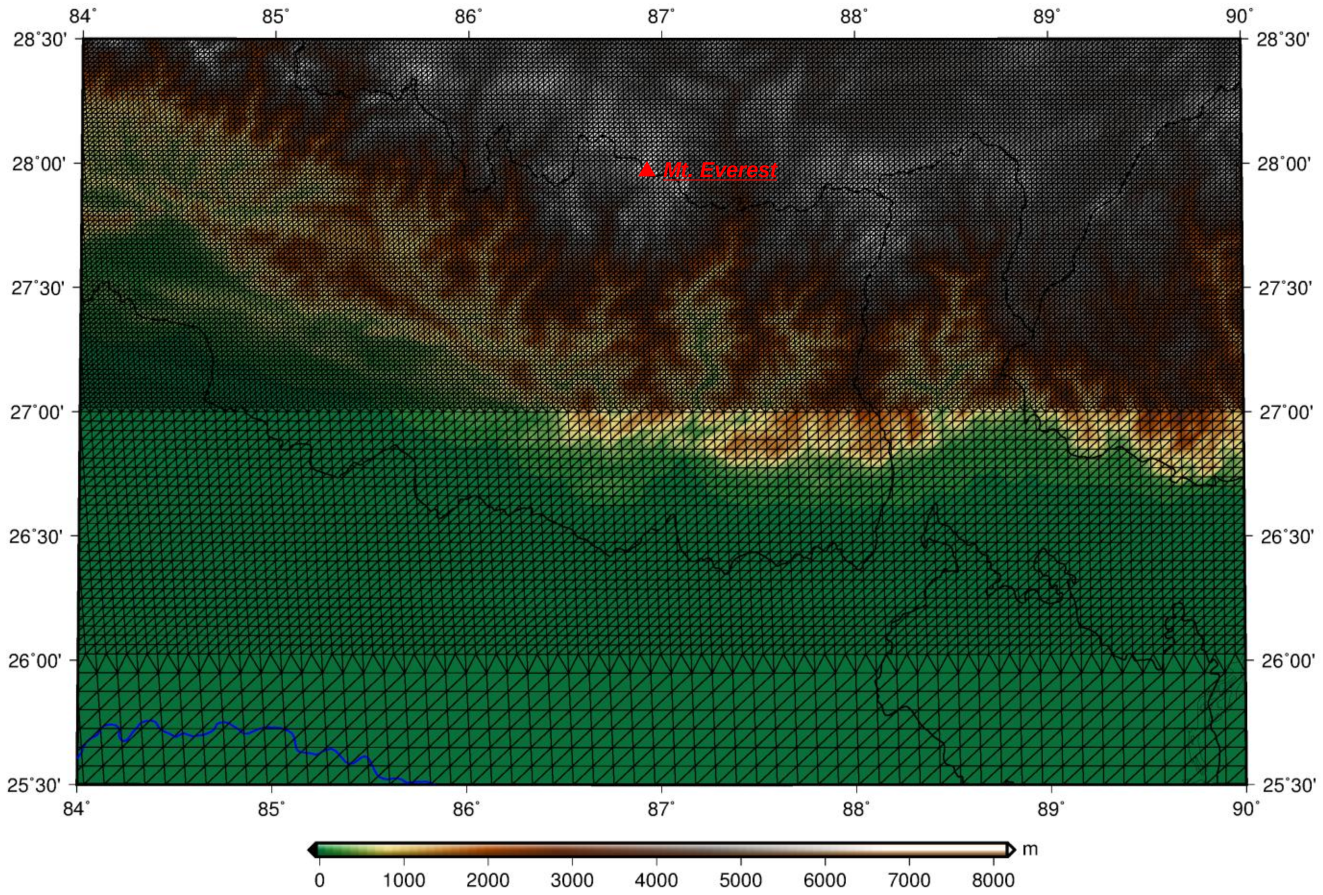






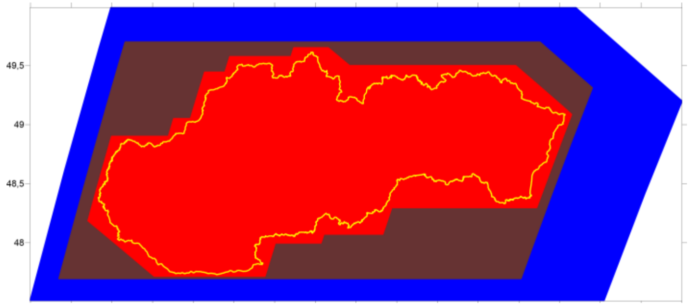


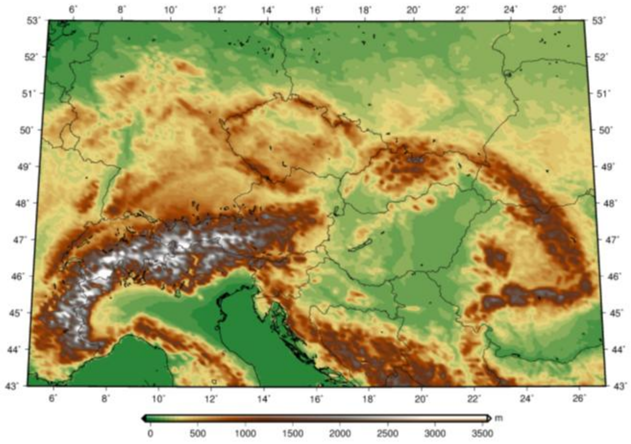
Local refinement of triangulation in Himalayas

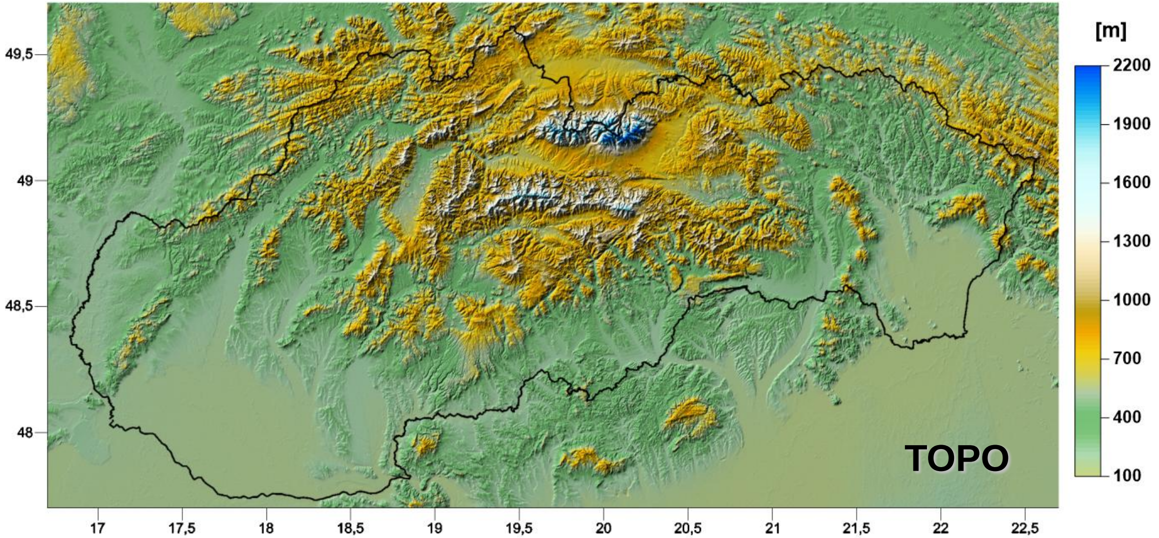


Local refinement in Slovakia (EU)

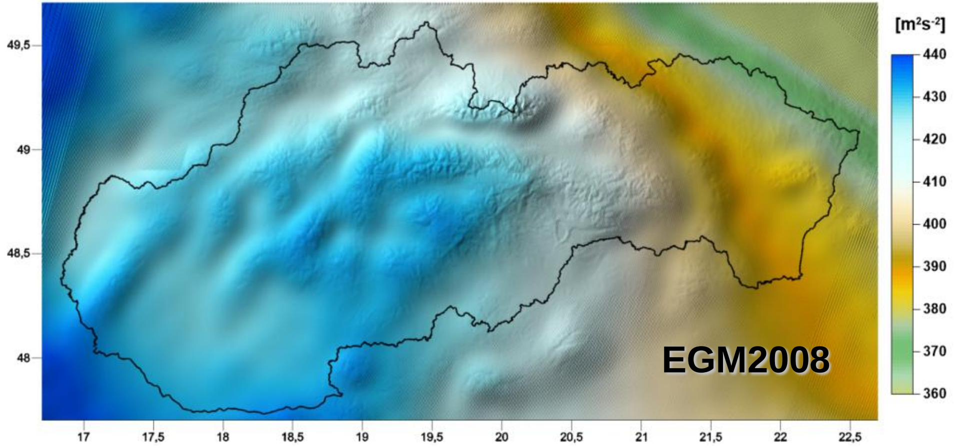
Global resolution: 0.075°
+ local refinement 1: 0.0375°
+ local refinement 2: 0.01875°
+ local refinement 3: 0.009375°
+ local refinement 4: 0.0046875°
+ local refinement 5: 0.00234375° (~*260 m*)

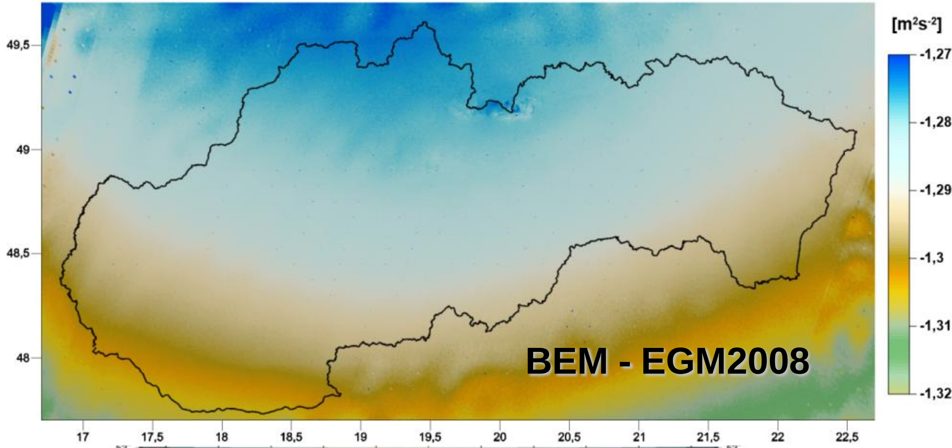


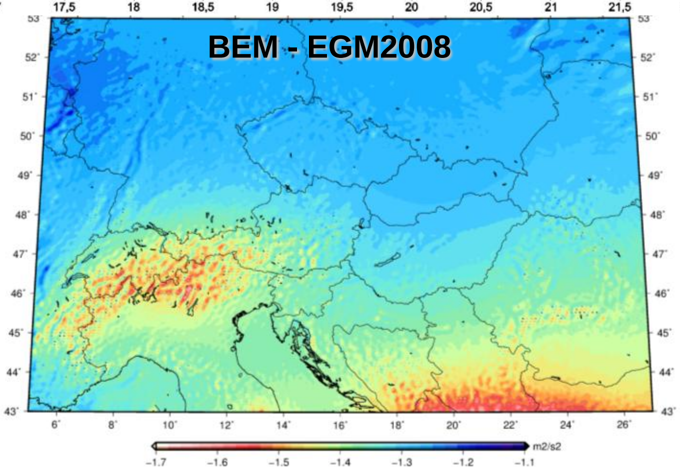




Reconstruction of EGM2008 in Slovakia



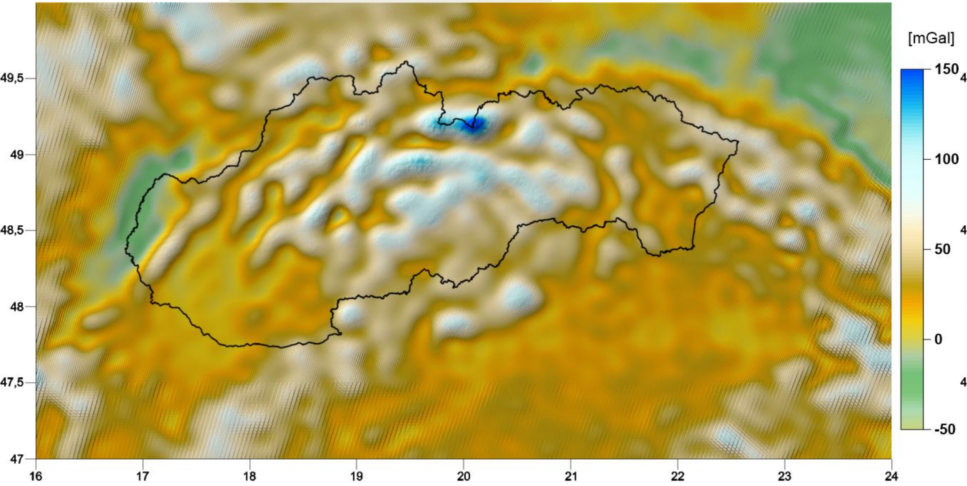




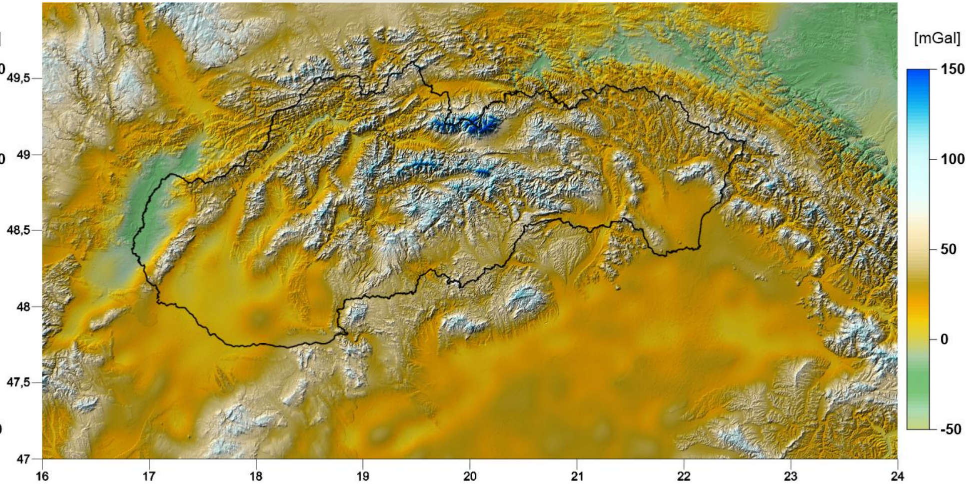
STATISTICS OF RESIDUALS	
Nodes	720 923
Mean	-1.288 m^2s^{-2}
Max	-1.261 m^2s^{-2}
Min	-1.352 m^2s^{-2}
STD	0.0093 m^2s^{-2}

Input surface gravity disturbances

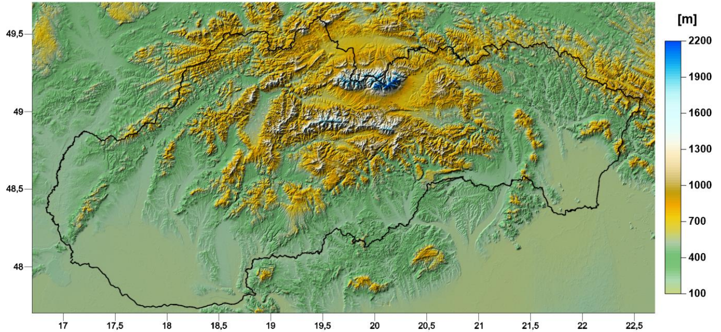
EGM2008 *(SH up to d/o 2160)*

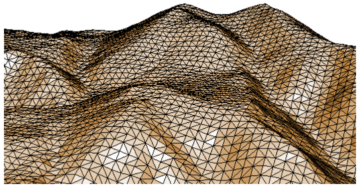


GGMplus *(EIGEN-6C4 + RTM)*



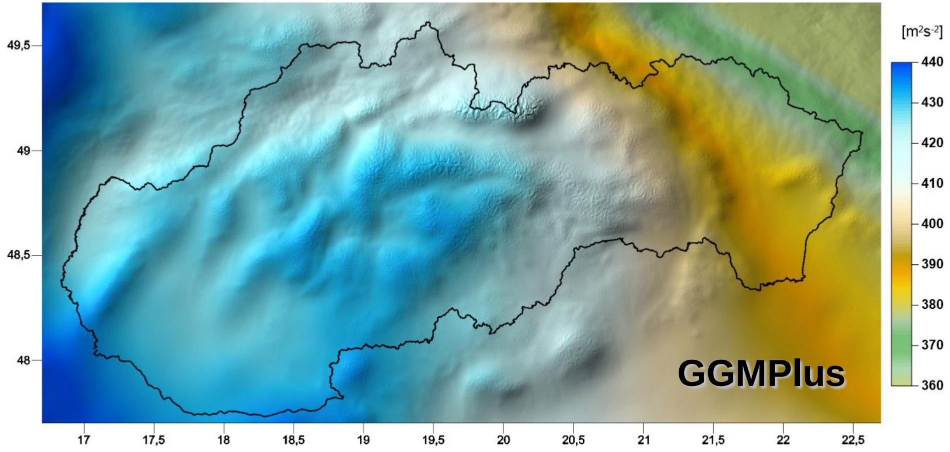
TOPO

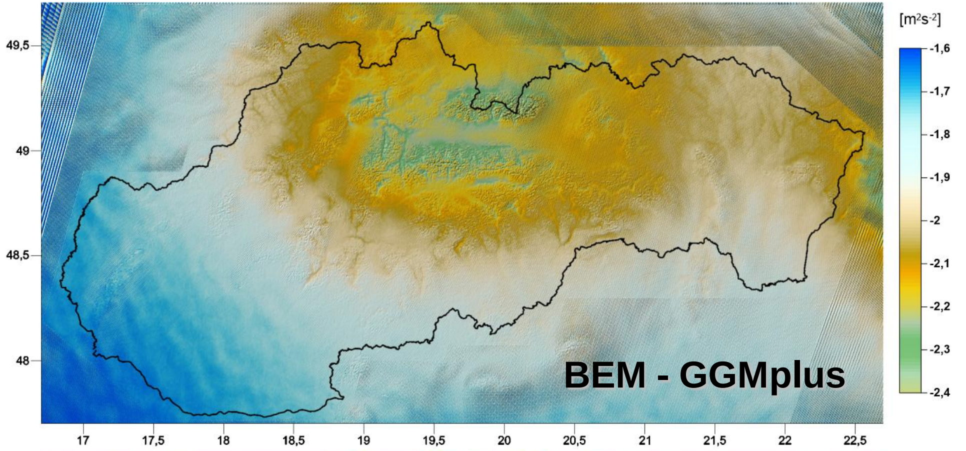




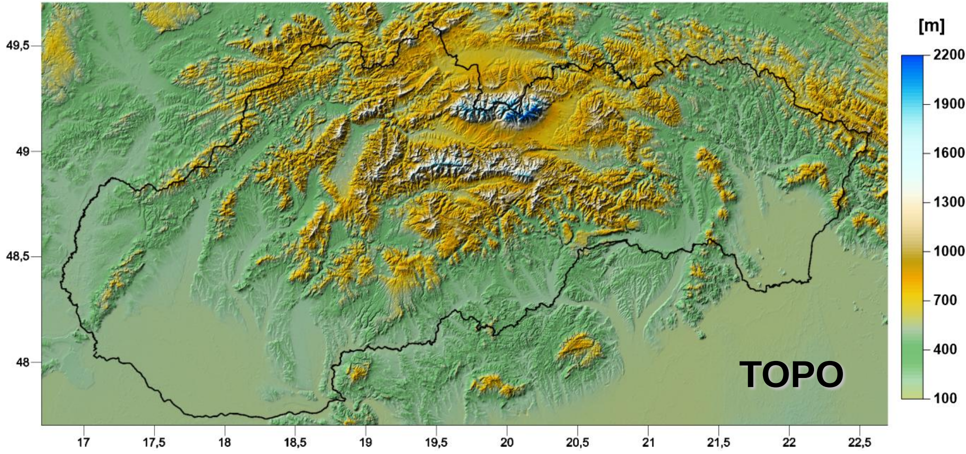
http://www.geom.at/fade2d/html/

Reconstruction of GGMPlus

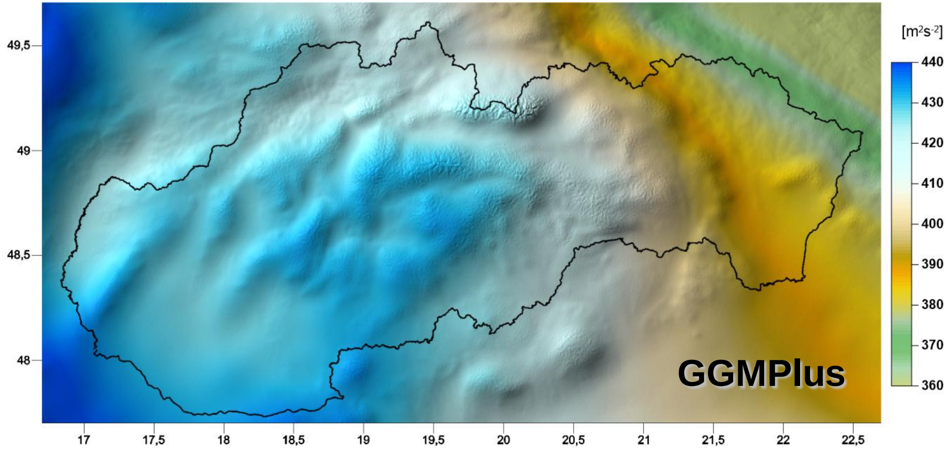


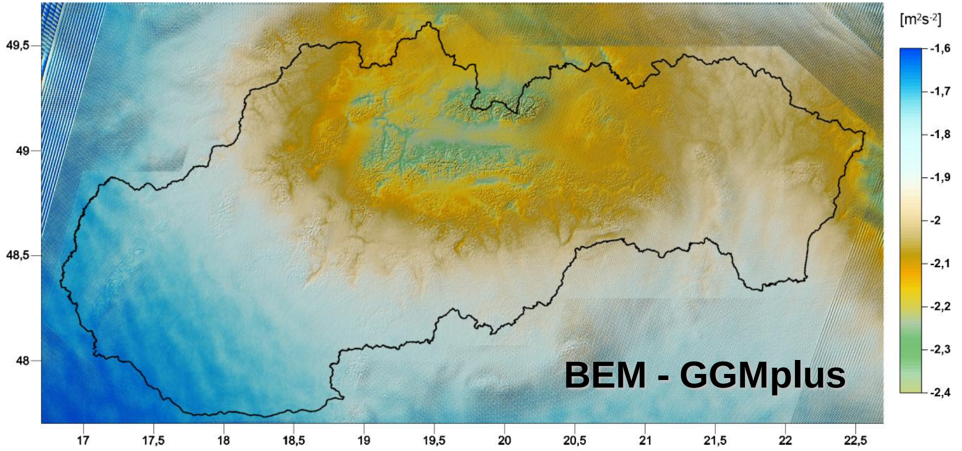


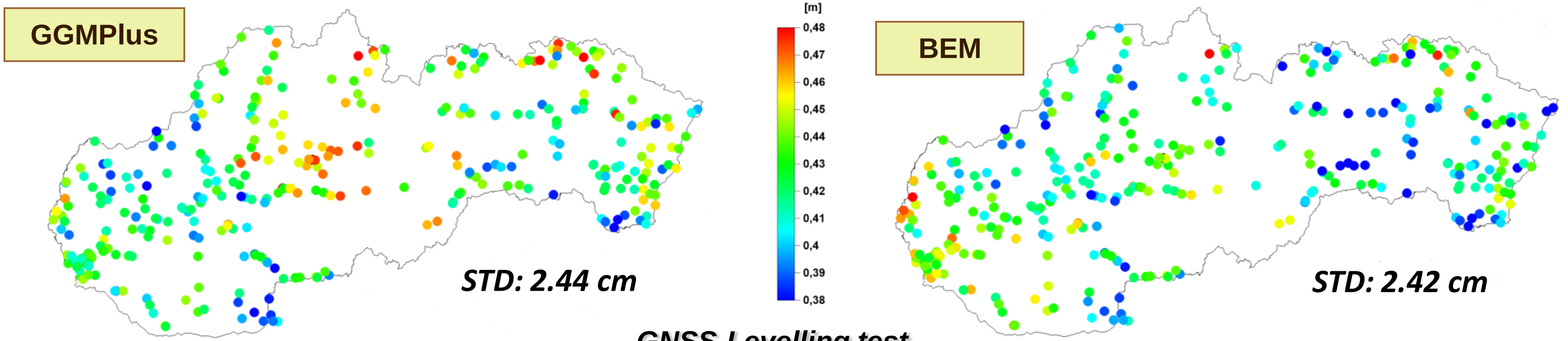
STATISTICS OF RESIDUALS	
Nodes	720 923
Mean	-2.285 m^2s^{-2}
Max	-1.062 m^2s^{-2}
Min	-3.721 m^2s^{-2}
STD	0.387 m^2s^{-2}



Reconstruction of GGMPlus

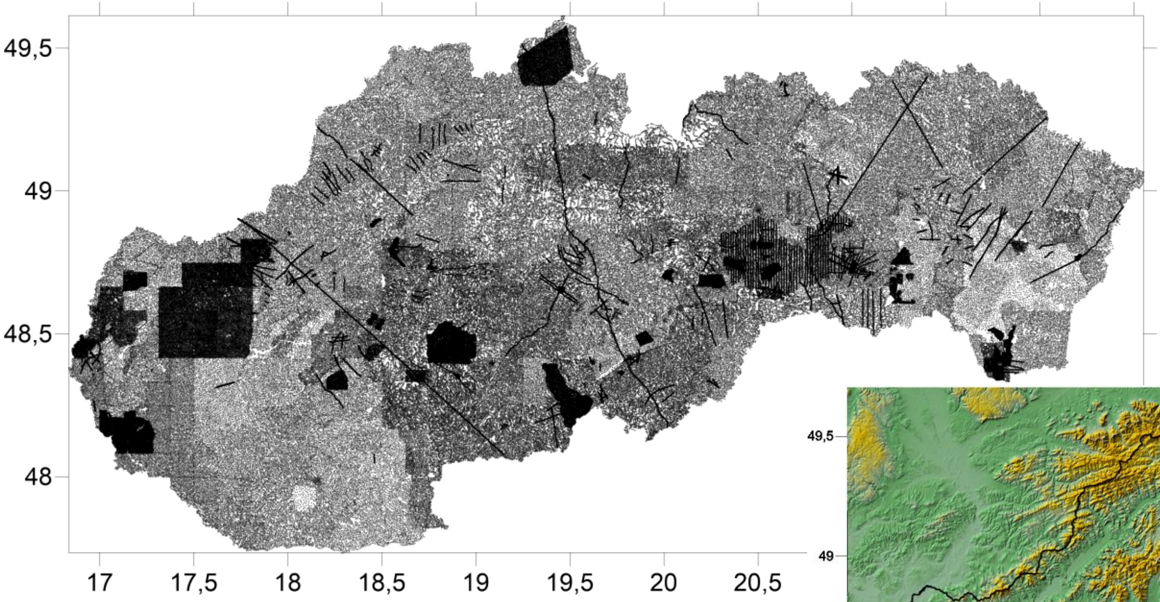






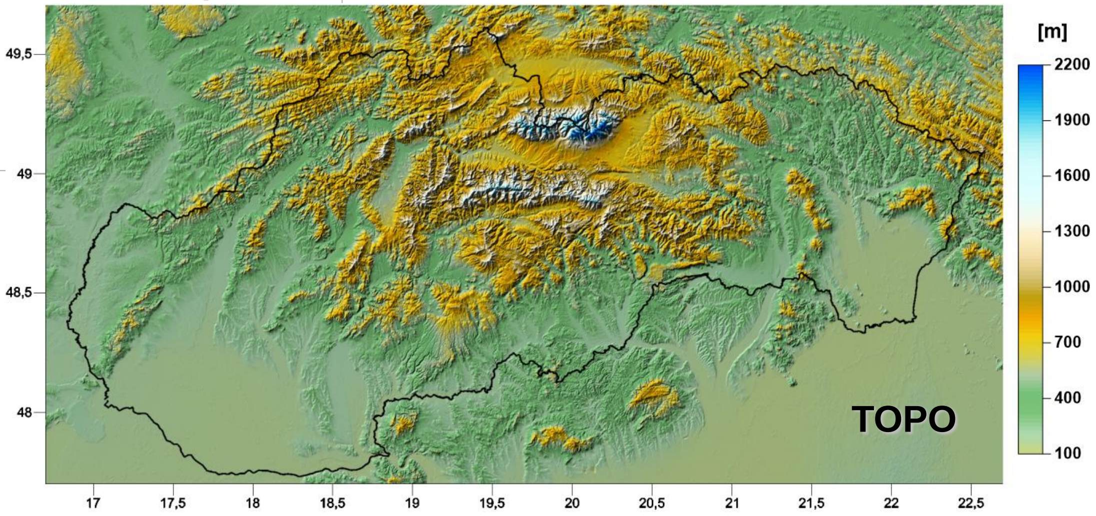
GNSS-Levelling test

Terrestrial gravimetric mapping in Slovakia



- **more than 220 000 measurements**
  (collected during the last decades)

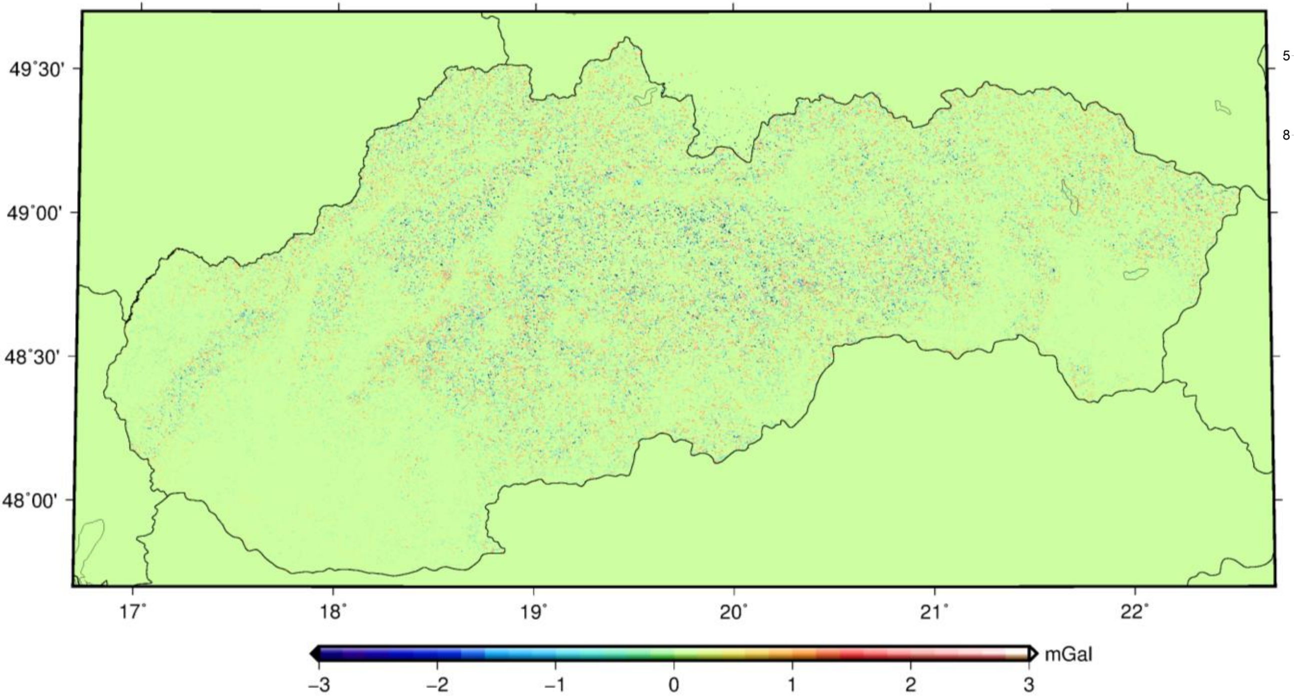
Digital terrain model

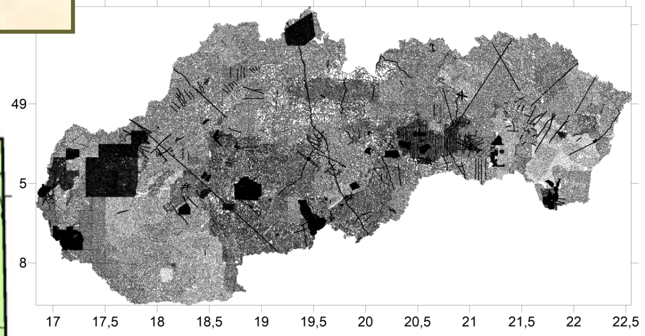


Terrestrial gravimetric measurements

"GGMplus nodes" replaced by original measurements: if (dist < 120 m)

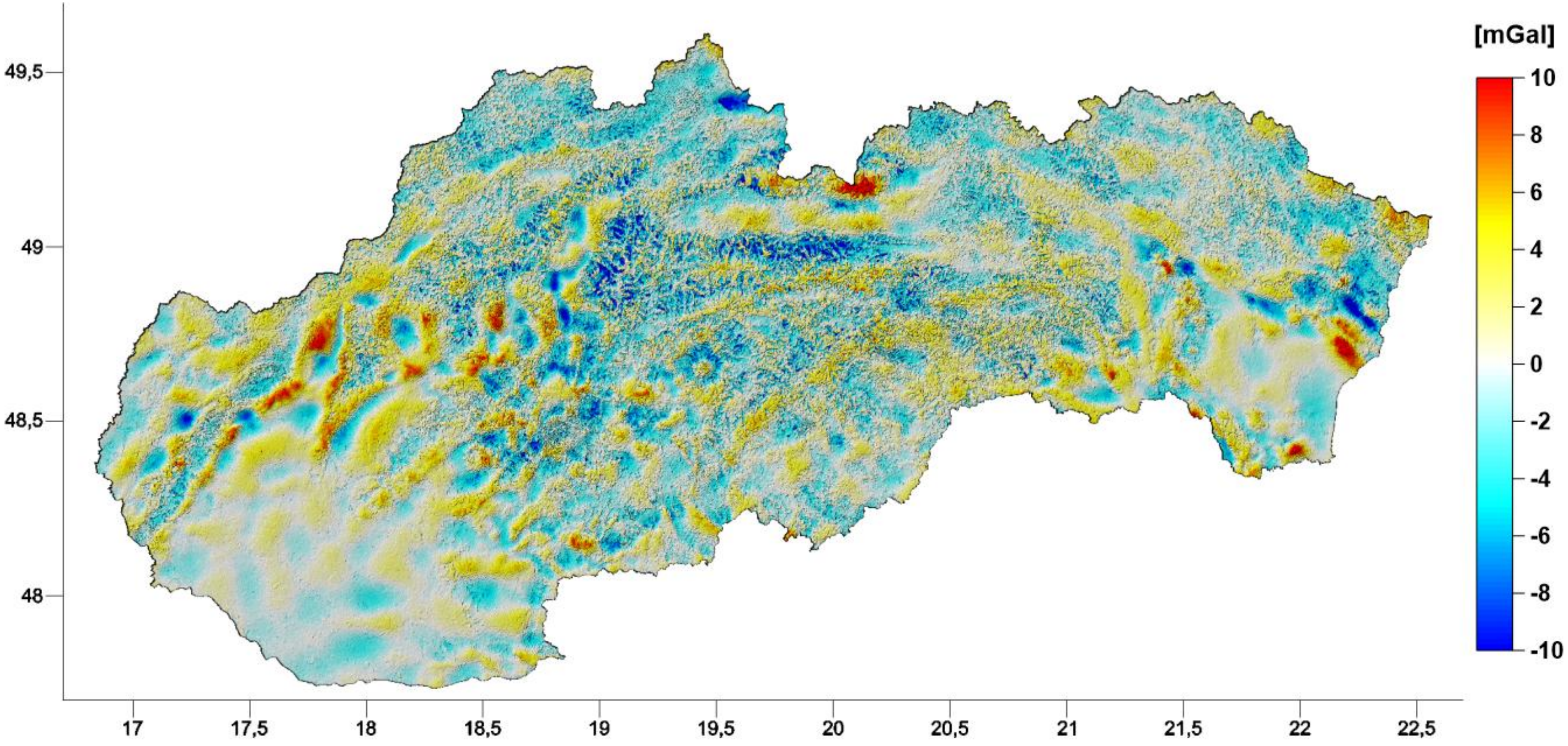
- ***about 55% of all nodes in Slovakia replaced***





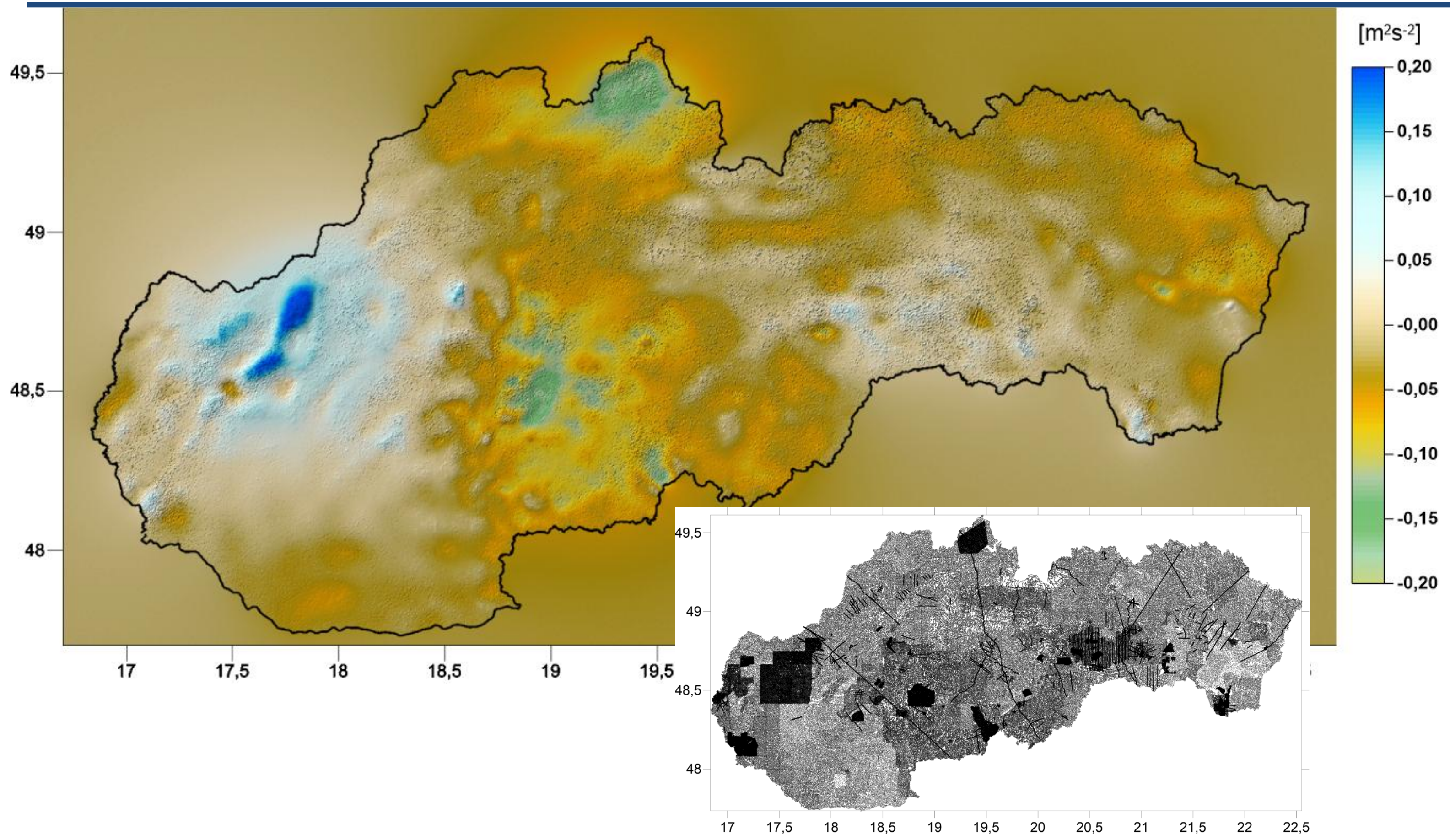
→ **modified triangulation**

Original gravimetric data – GGMPlus

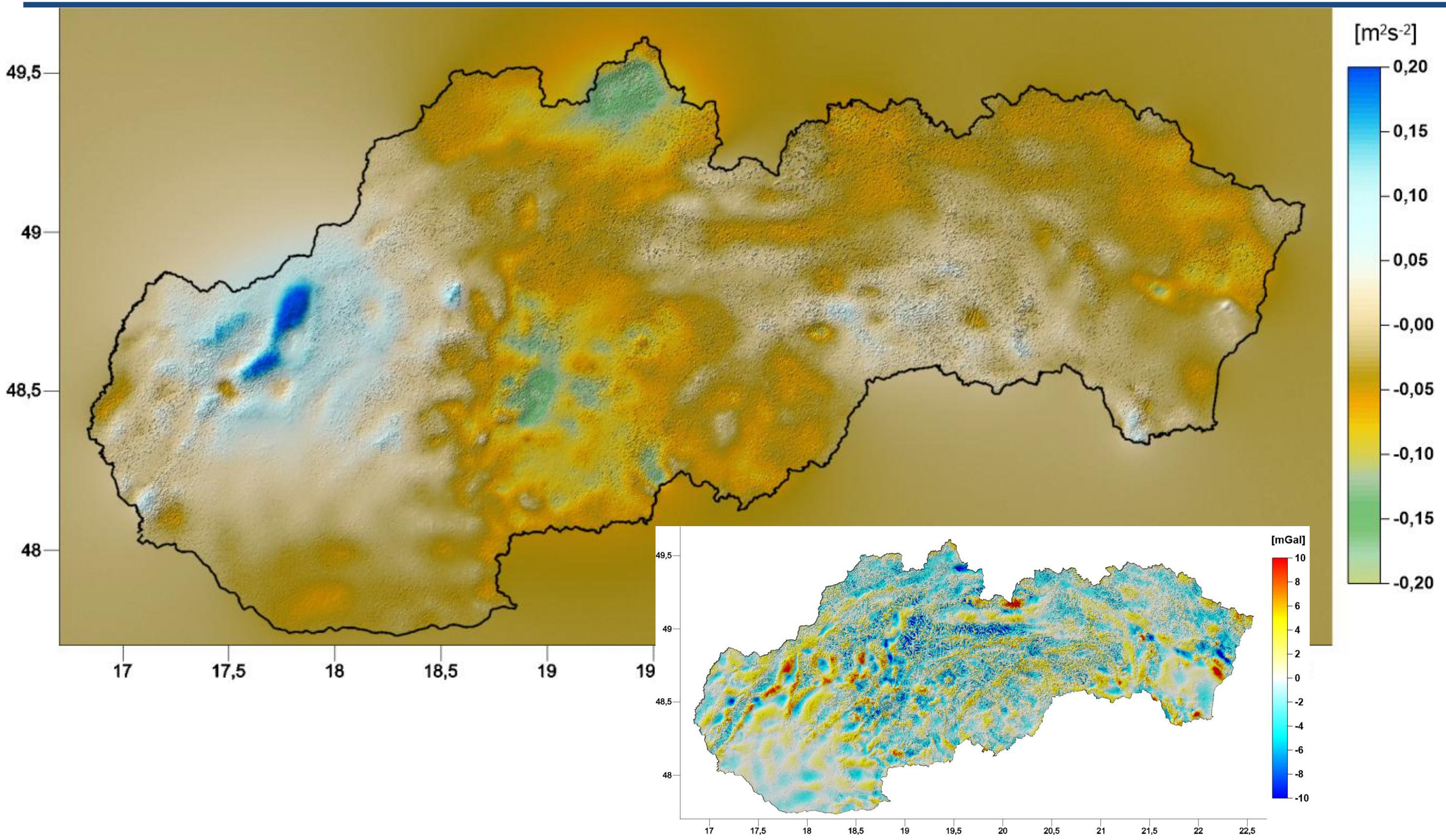


!!! Local extremes exceed ±10 mGal !!!

Contribution of gravimetric measurements



Contribution of gravimetric measurements



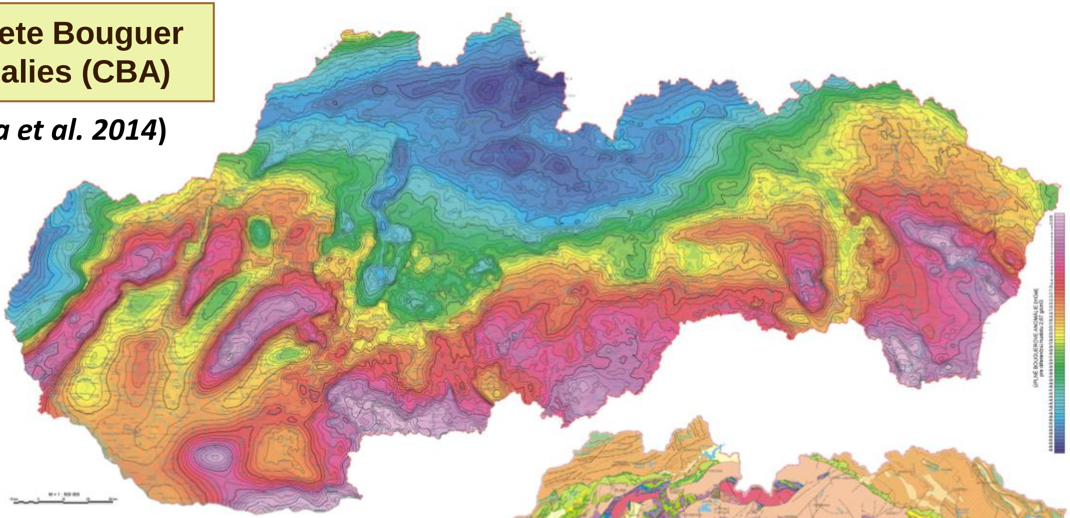
Input gravity disturbances generated from CBA

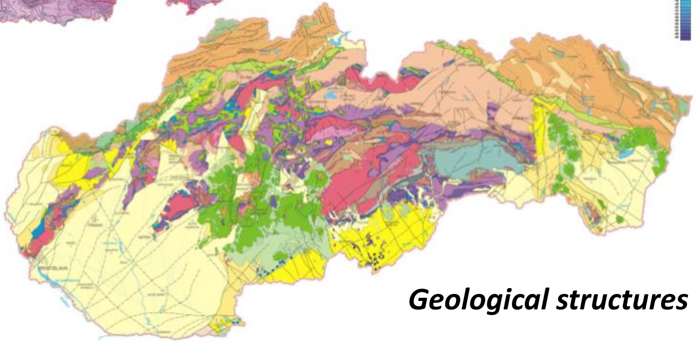
Remained "GGMplus nodes" replaced by generated gravity disturbances

- *about 45% of all nodes in Slovakia*

Complete Bouguer Anomalies (CBA)

(*Pašteka et al. 2014*)





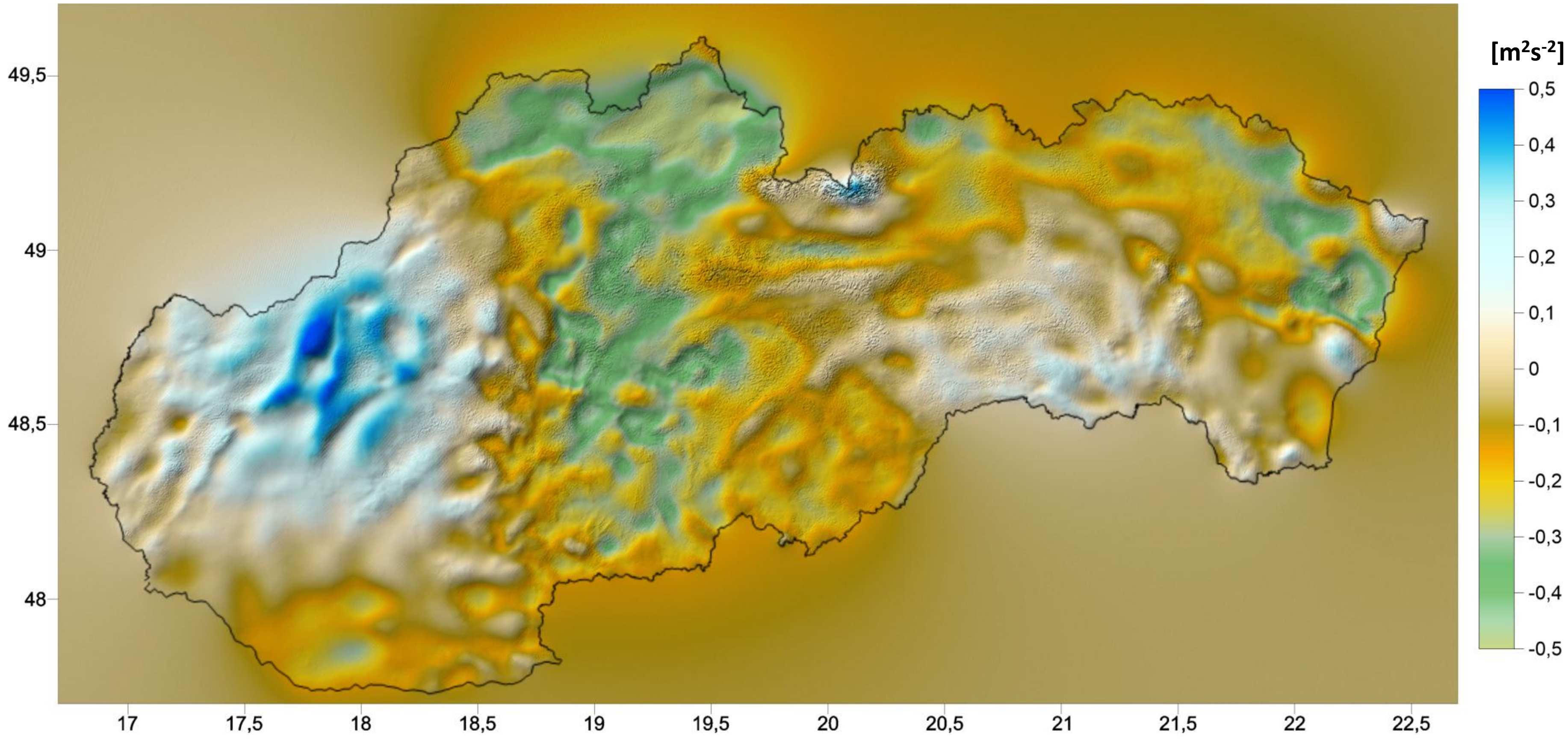
Geological structures

CBA2G_SK software

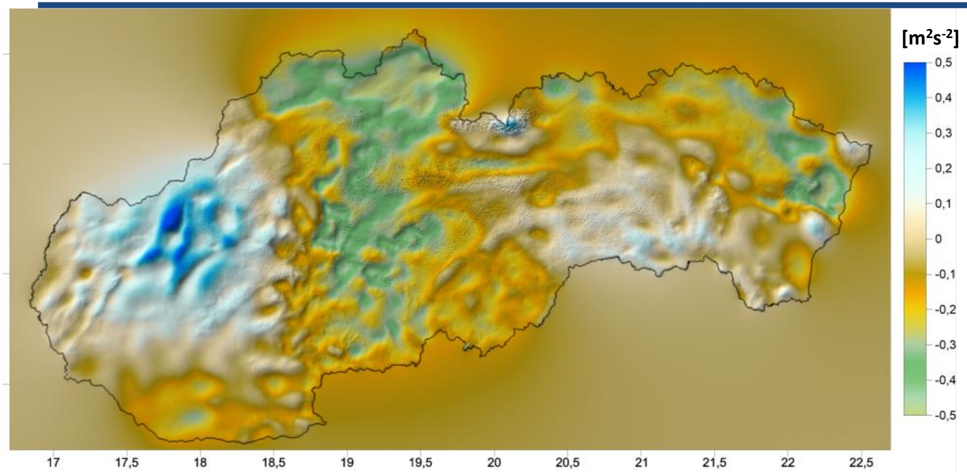
(*Marušiak et al. 2015*)

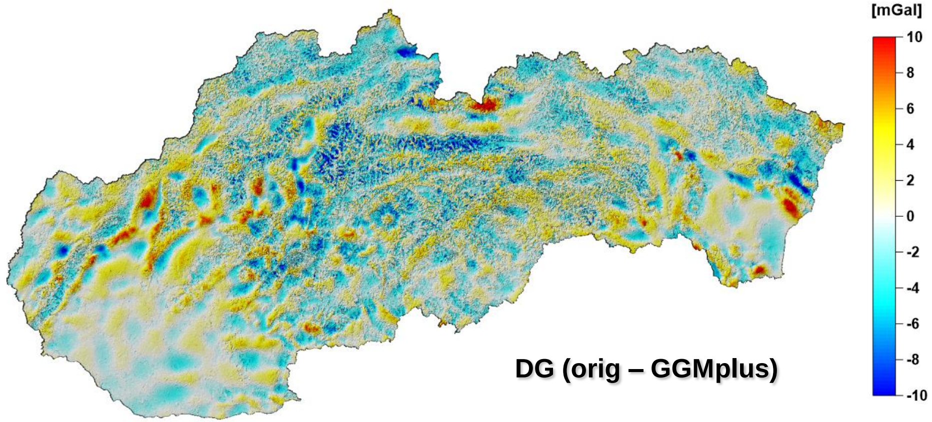
$$g_{CBA2G} \doteq \Delta g_{CBA} + \gamma_0 + \delta g_{faa} + \\ + \delta g_{sph}^{0-166.7\,km} - \delta g_{atm} - \delta g_{top}^{0-166.7\,km}$$

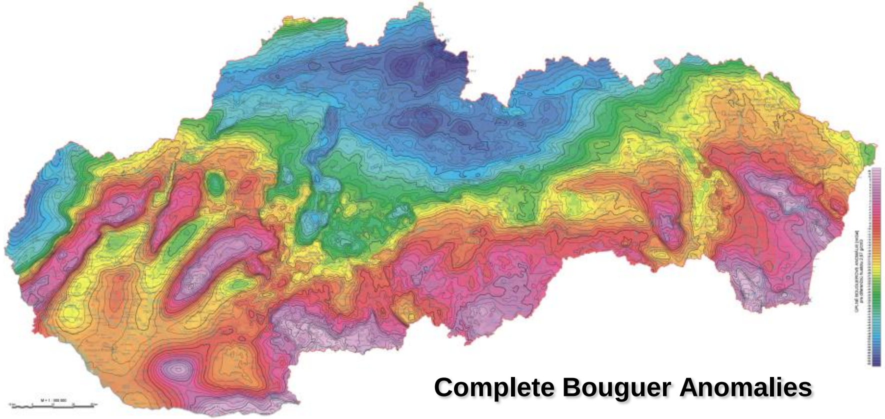
Contribution of terrestrial + generated from CBA



Correlation with Bouguer anomalies?







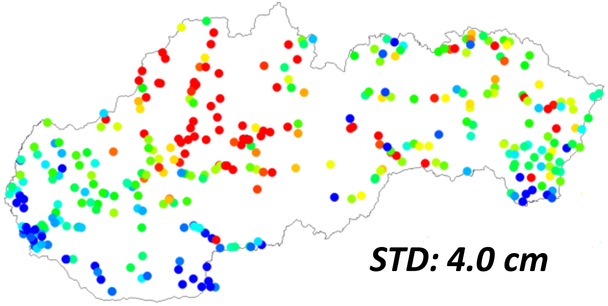
??? Open questions ???

- How is the impact of the low-frequency part of the GOCE-based GGMs (~ 2 cm accuracy)?
- Can we really detect ‘biases’ in the terrestrial gravimetric measurements in ‘low frequencies’?
- How does it influence the quasigeoid modelling (e.g. using the R-C-R strategies)?

GNSS-Levelling test of combined GGMs in Slovakia

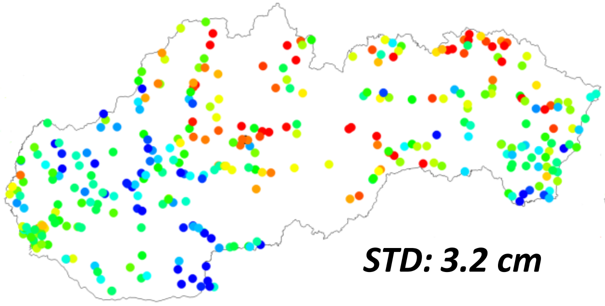
EGM-2008

(SH up to d/o 2160)



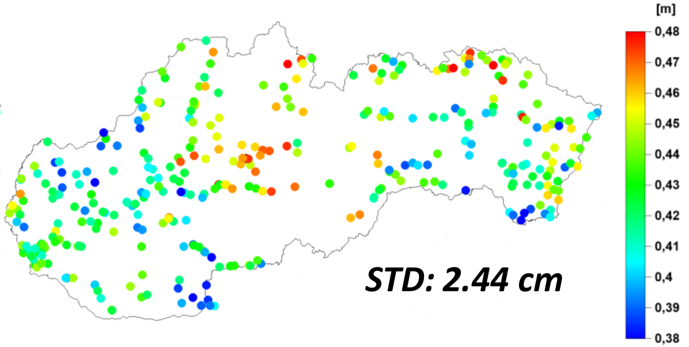
EIGEN-6C4

(SH up to d/o 2160)



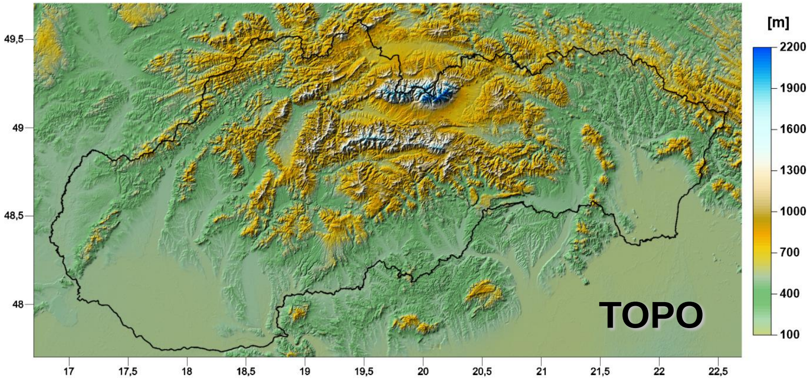
GGMPlus

(EIGEN-6C4 + residual terrain model)



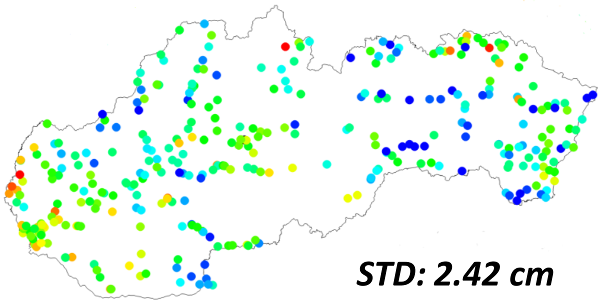
- ***at 336 benchmarks***

provided by
GKÚ Bratislava
(10 outliers removed)



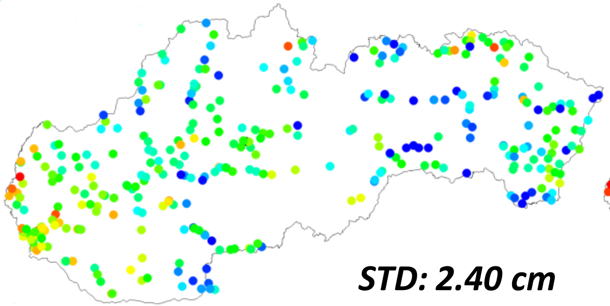
GNSS-Levelling test of BEM solutions

Reconstruced GGMPlus



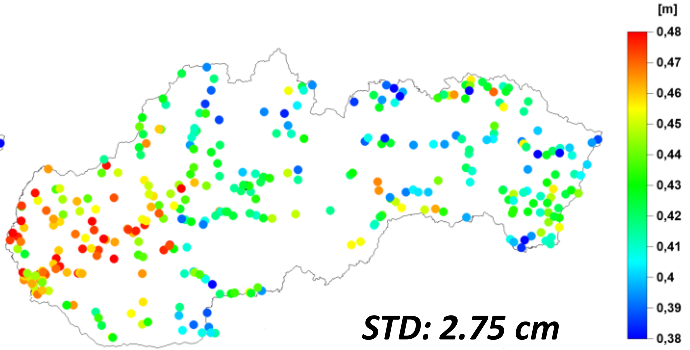
STD: 2.42 cm

… + original gravity data

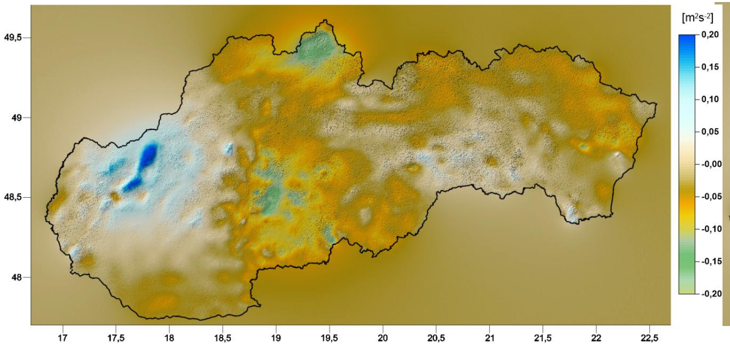


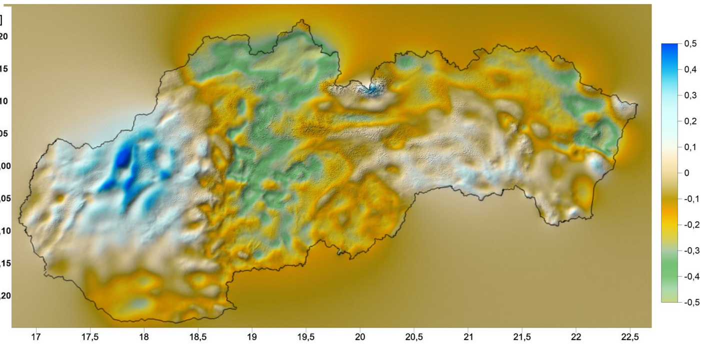
STD: 2.40 cm

… + generated from CBA



STD: 2.75 cm





Possible contribution for realization of IHRS

Advantages

- BEM solution is obtained directly at points on the Earth's surface

  → *terrestrial gravimetric measurements can directly represent computational nodes considering their 3D positions*

  → *there is no need to make any reduction from masses or heights!!!*

- Local BEM solutions can reach "cm-level" accuracy

  → *requires very dense distribution of terrestrial gravimetric data*

  → *achieved precision is dependent on quality of input data*

Drawbacks

- BEM solutions is biased due to an insufficient global discretization

  → *this can be overcome by a reconstruction of a known harmonic function (e.g. EGM-2008) on a same computational grid → this yields "the correction function from the discretization error"*

Realization of IHRS (concept)

1) Vertical coordinates are potential differences with respect to a conventional W_0 value:
   - $C_P = C(P) = W_0 - W(P) = -\Delta W(P)$
   - conventional fixed value $W_0 = const. = 62\ 636\ 853.4\ \mathrm{m^2s^{-2}}$
2) The position P is given by the coordinates vector $\mathbf{X}_P$ (X_P, Y_P, Z_P) in the ITRF; i.e., $W(P) = W(\mathbf{X}_P)$
3) The estimation of $\mathbf{X}(P)$, $W(P)$ (or $C(P)$) includes their variation with time; i.e., $\dot{\mathbf{X}}(P)$, $\dot{W}(P)$ (or $\dot{C}(P)$).
4) Coordinates are given in mean-tide system / mean (zero) crust.

Conclusions

- *Global approach based on precise gravity field modelling*
  is suitable for a realization of the Vertical Reference Systems (also on continental scale)

  → *globally consistent*

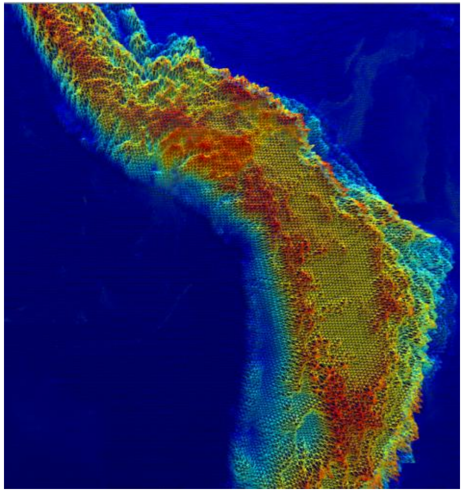
  → *at present, ”cm-level” accuracy can be achieved by a combination of the combined GGMs (e.g. EGM-2008, EIGEN-6C4) with residual terrain model (e.g. GGMPlus), however, precise local (national) quasigeoid modelling can lead to more precise solutions (if terrestrial or airborne gravity data are available)*

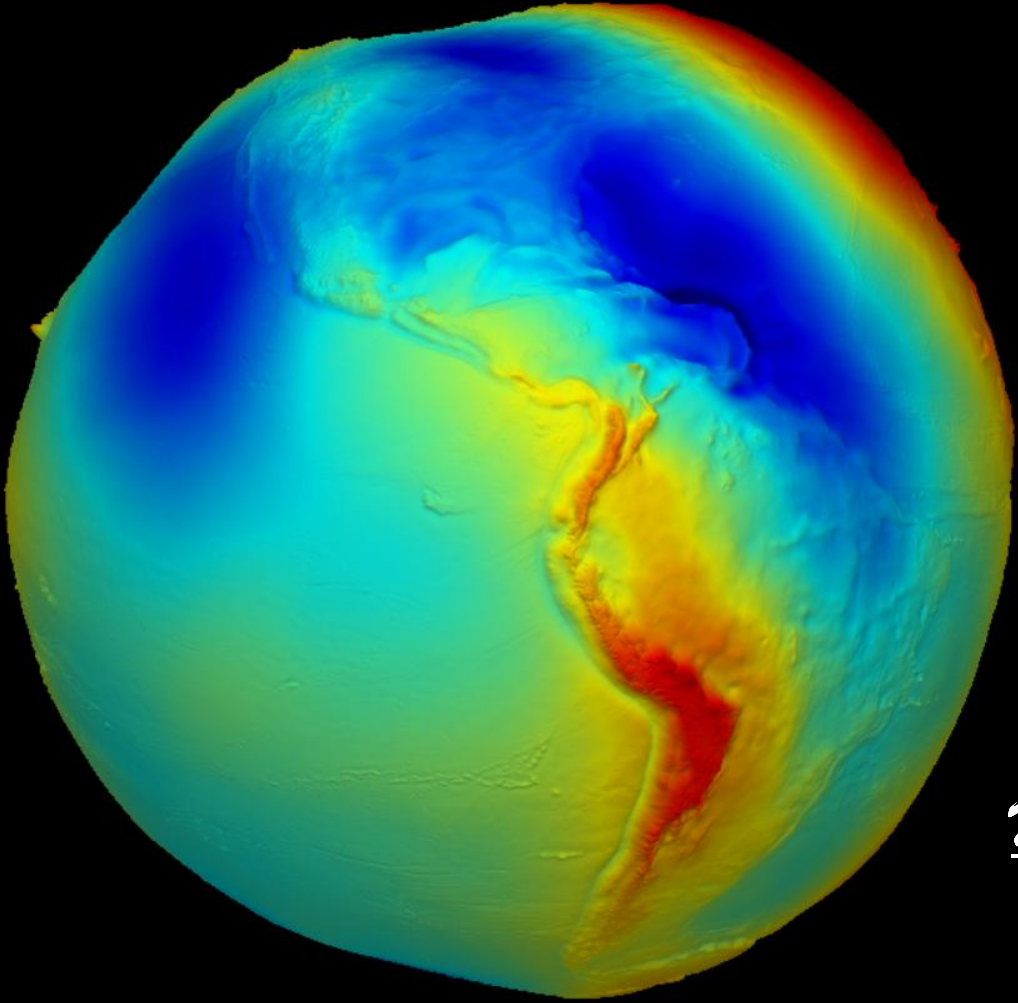
  (Remark: quasigeoid is nothing else than the disturbing potential on the Earth’s surface rescale to metric units)

- *Fixed gravimteric BVP should be preferred*

  → *input gravity disturbances are independent from local LVDs (globally consistent)*

- *BEM approach allows to determine geopotential on the Earth’s surface*





Muchas gracias
por la atención