



SIRGAS

Sistema de Referência Geocêntrico
para as Américas

2019

Rio de Janeiro,
Brasil

An Analysis of the Use of Least Squares Collocation and the Numerical Integration to Compute the Disturbing Potential at IHRF Stations in Brazil

Uma Análise do uso da Colocação por Mínimos Quadrados e da Integração Numérica para o cálculo do Potencial Perturbador nas Estações IHRF no Brasil



Gabriel do Nascimento Guimarães ⁽¹⁾

(gabriel@ufu.br)

Denizar Blitzkow ^(2,3)

(dblitzko@usp.br)

Ana Cristina Oliveira Cancoro de Matos ⁽²⁾

(acocmatos@gmail.com.br)

⁽¹⁾ Universidade Federal de Uberlândia, Brazil

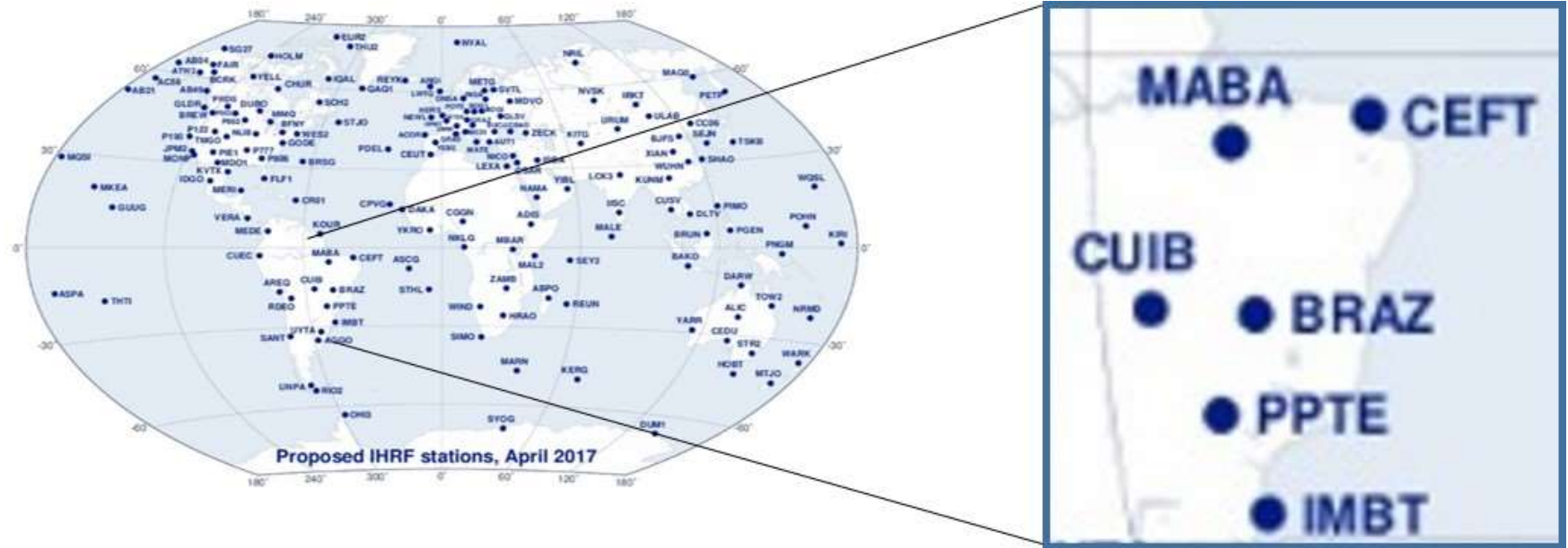
⁽²⁾ Centro de Estudos de Geodesia, Brazil

⁽³⁾ Universidade de São Paulo, Brazil



- ☐ Motivation
- ☐ Gravimetric and Topographic Analysis
- ☐ Disturbing Potential Computation
- ☐ Results
- ☐ Remarks and Next Steps

Brazilian IHRF stations



GNSS Brazilian Continuous Monitoring Network (RBMC)

Fortaleza (CEFT) → March 2010

Marabá (MABA) → September 2007

Cuiabá (CUIB) → January 2000

Brasília (BRAZ) → March 1995

Presidente Prudente (PPTE) → December 1996

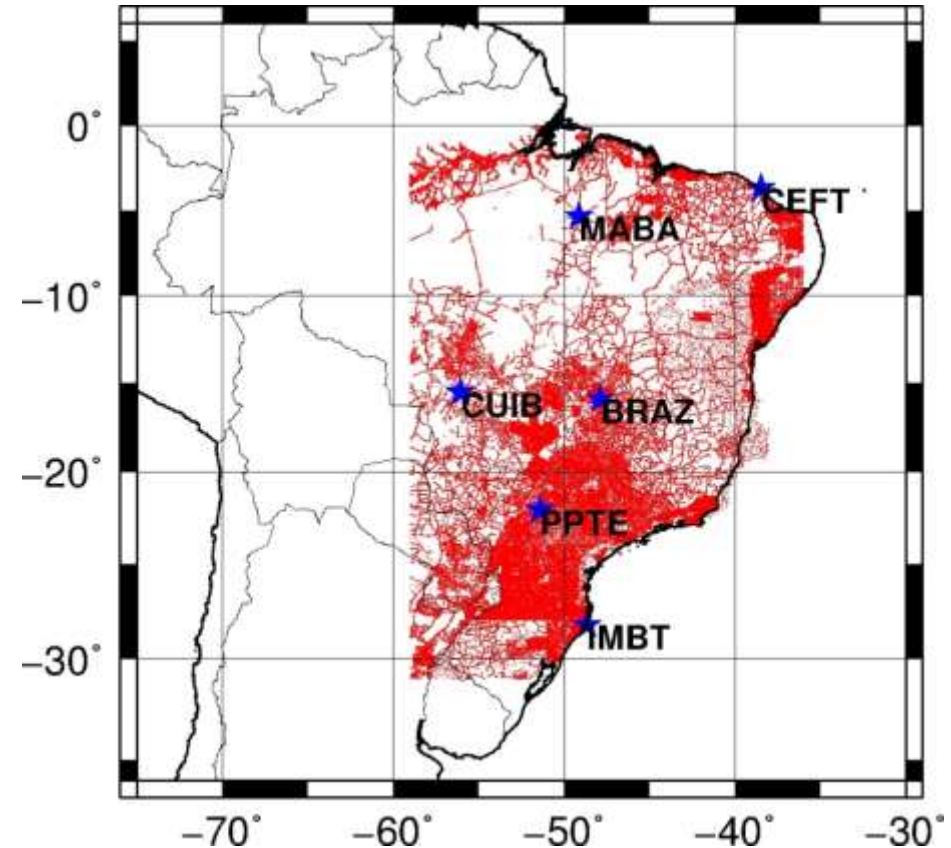
Imbituba (IMBT) → September 2007



Brazilian Tide Gauge

Terrestrial Gravity Distribution

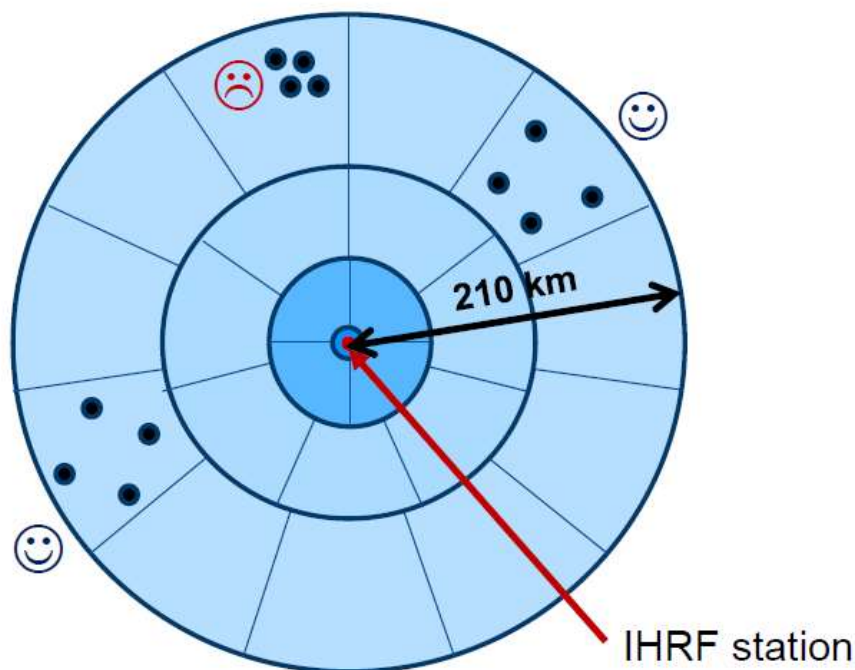
- 385,649 gravity stations in a region of 0° S to 31° S in latitude and 59° W a 36° W in longitude.
- 3 stations with absolute gravity (CUIB, BRAZ, PPTE).



- Brazilian Institute of Geography and Statistics (IBGE);
- Center for Geodesy Studies (CENEGEO);
- South America Gravity Studies* (SAGS);
- National Observatory (ON);
- Oil and gas National Agency (ANP);
- Nationals universities.

CUIB

BRAZ



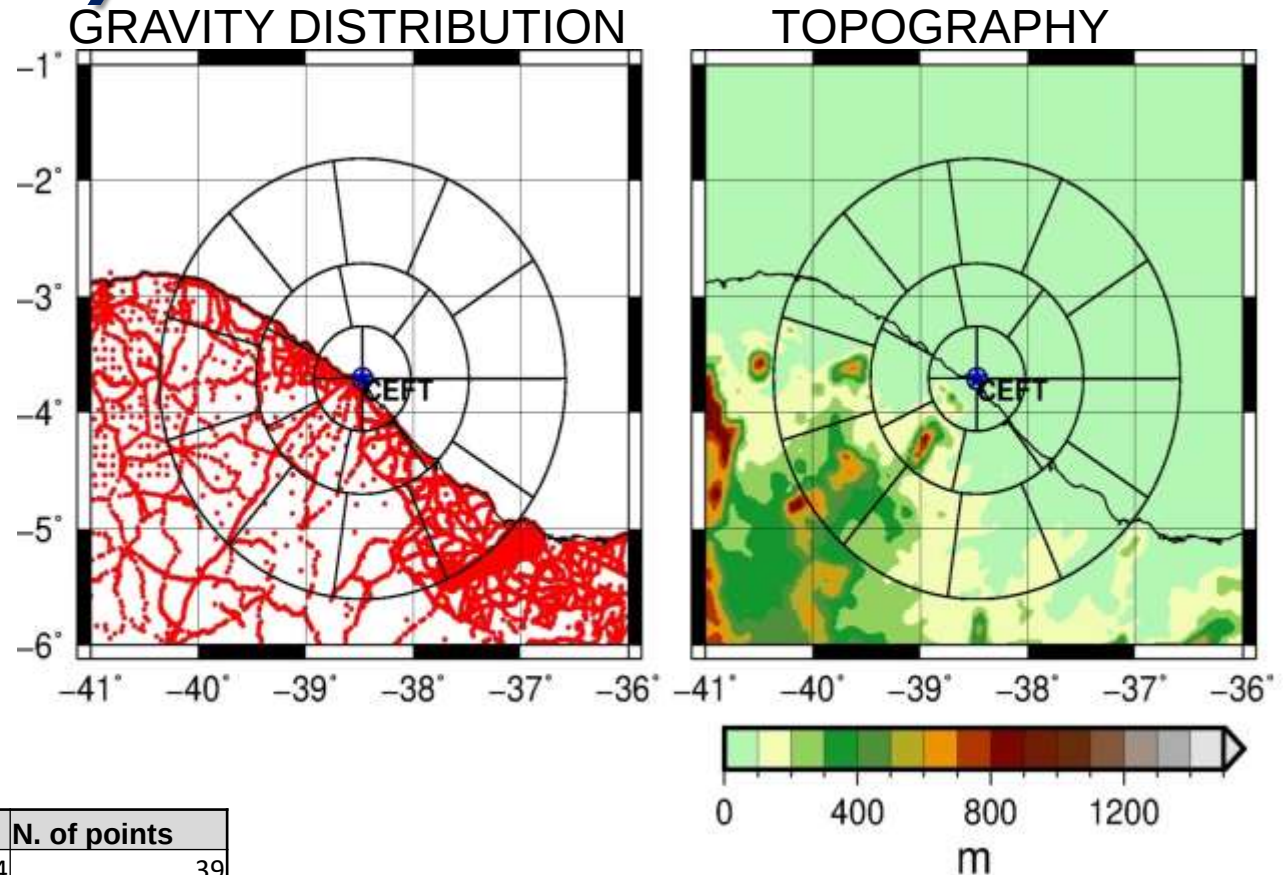
Template according to the gravity effect on the geoid ($\Delta g = 1 \cdot 10^{-6} \text{ ms}^{-2} \rightarrow 1 \text{ mm}$)

Distance	Compartments	# of points flat/mountain
10 km	1	4/8
50 km	4	20/30
110 km	7	30/45
210 km	11	50/75
Sum	23	100/150

Towards a first realization of the International Height Reference System (IHRF)

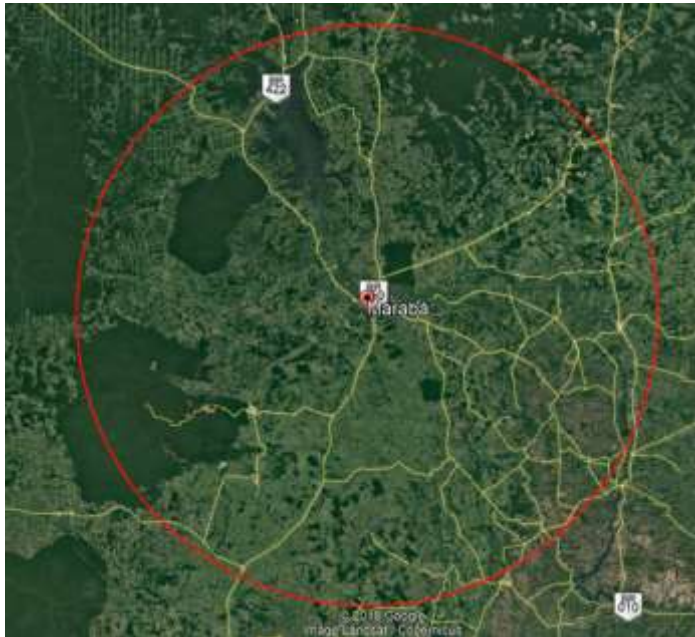
Laura Sánchez, Johannes Ihde, Roland Pail, Thomas Gruber, Riccardo Barzaghi, Urs Marti, Jonas Ågren, Michael Sideris, Pavel Novák.
European Geosciences Union General Assembly 2017. Vienna, Austria. April 25, 2017

Fortaleza (CEFT)

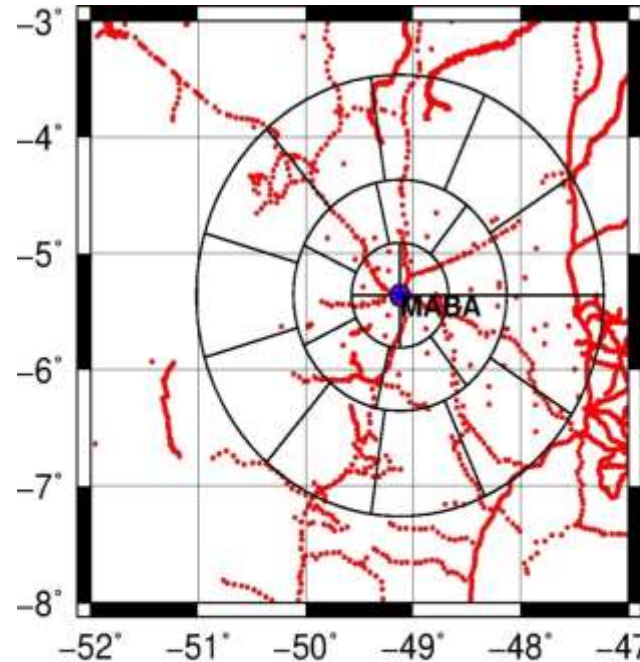


Stations	Distance	H mean	N. of points
CEFT	10 km	18.24	39
	10 km a 50 km	26.95	582
	50 km a 110 km	55.77	1104
	110 km a 210 km	74.11	4503
TOTAL OF POINTS			6228

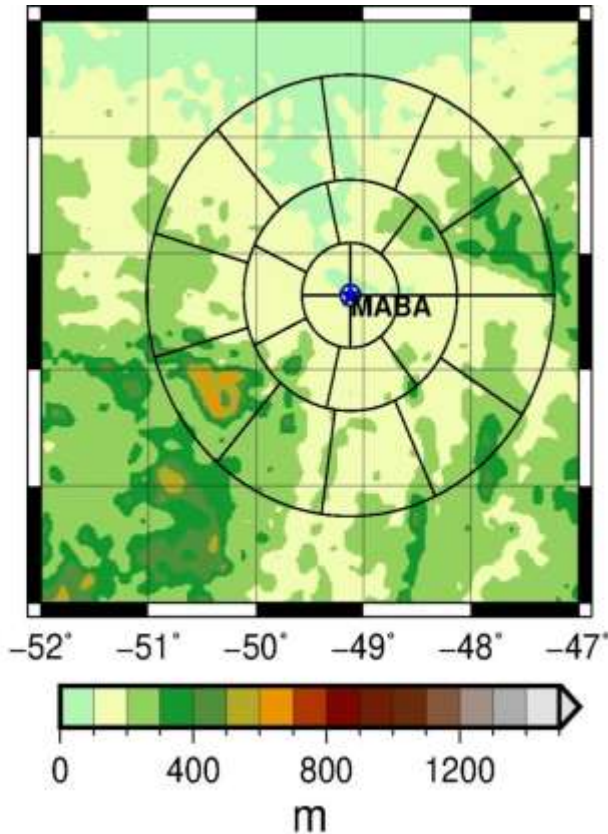
Marabá (MABA)



GRAVITY DISTRIBUTION

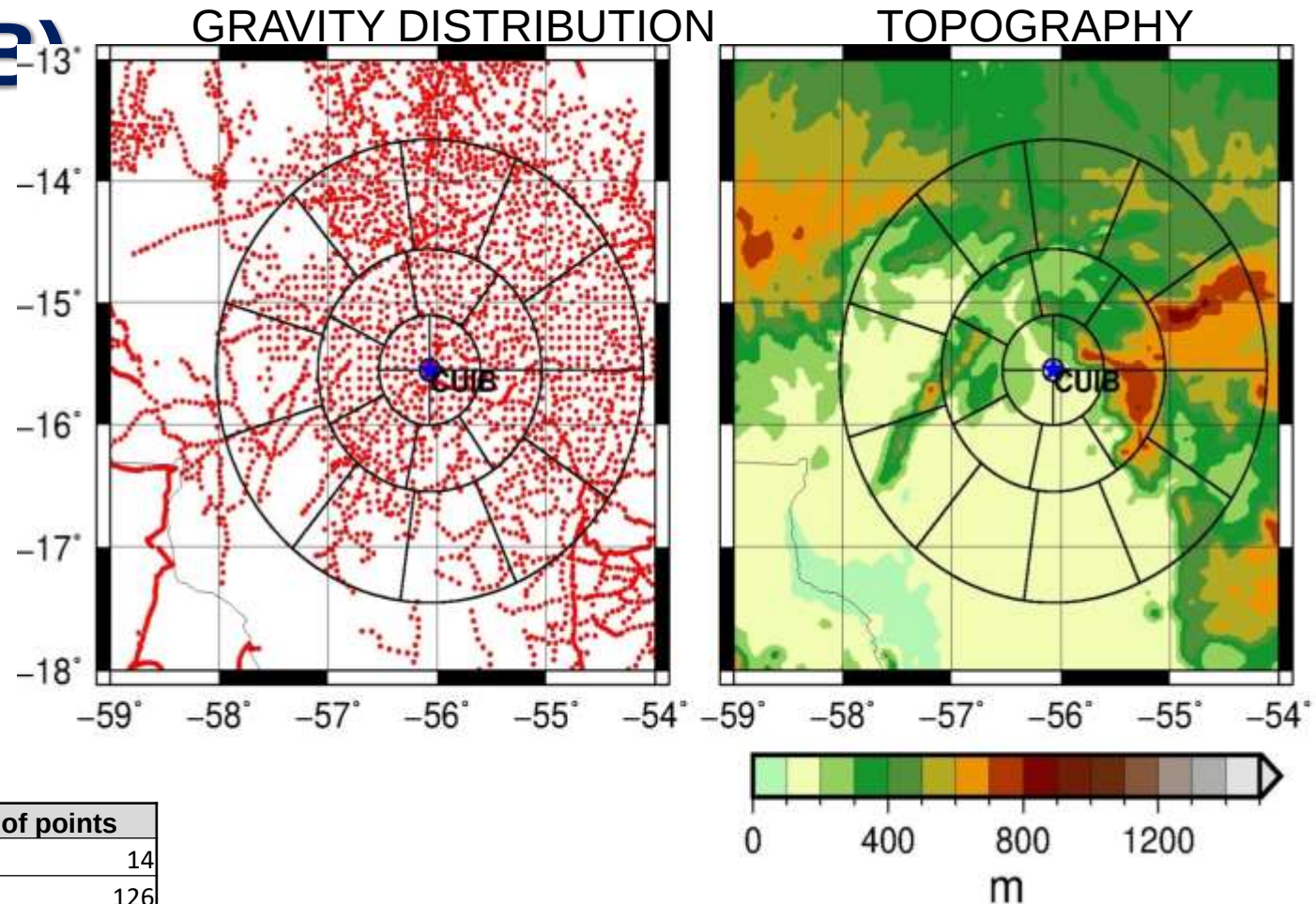


TOPOGRAPHY



Stations	Distance	H mean	N. of points
MABA	10 km	103.56	25
	10 km a 50 km	116.87	122
	50 km a 110 km	154.24	210
	110 km a 210 km	148.07	1101
TOTAL OF POINTS			1458

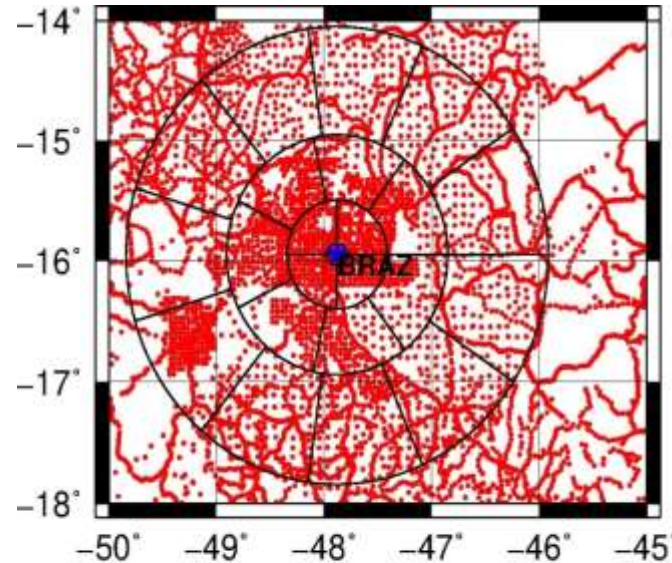
Cuiabá (CUIB)



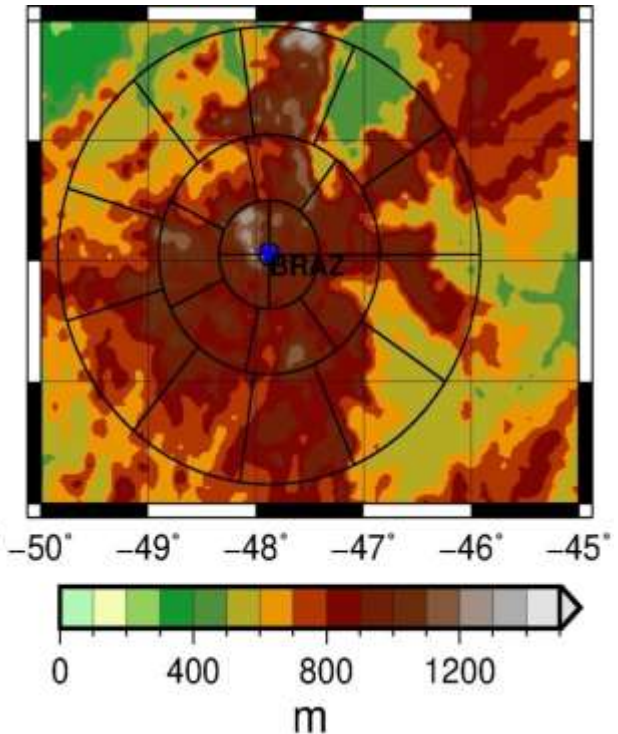
Stations	Distance	H mean	N. of points
CUIB	10 km	174.31	14
	10 km a 50 km	266.30	126
	50 km a 110 km	331.56	435
	110 km a 210 km	370.04	1323
TOTAL OF POINTS			1798

Brasília (BRAZ)

GRAVITY DISTRIBUTION

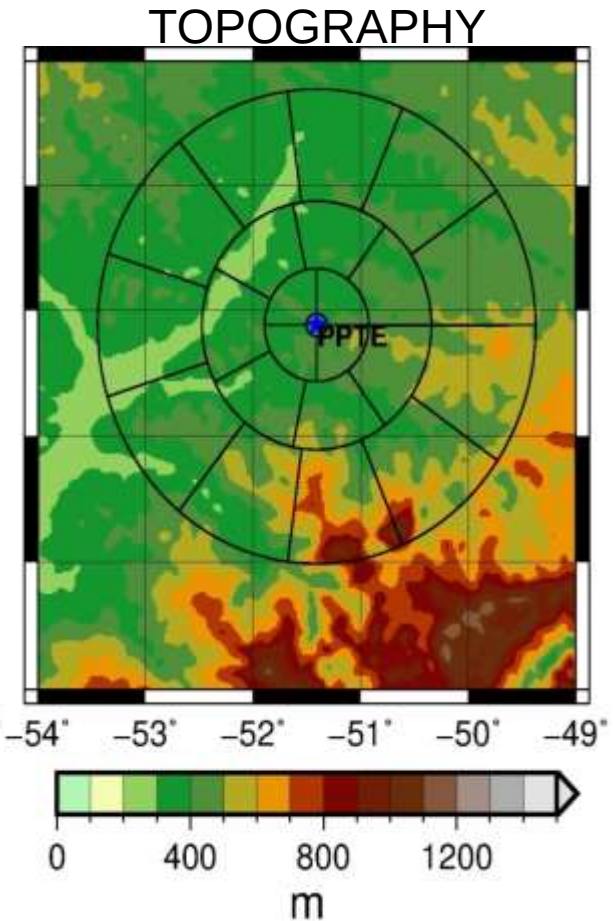
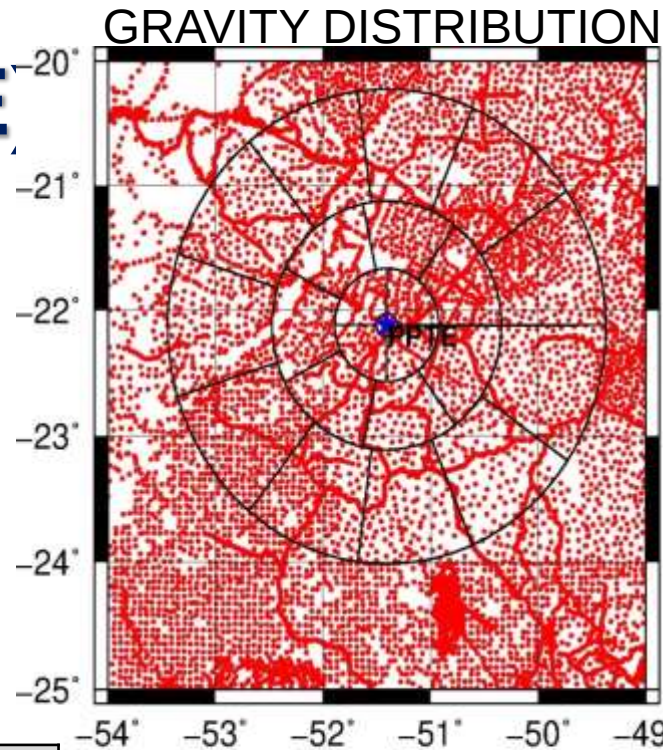


TOPOGRAPHY



Stations	Distance	H mean	N. of points
BRAZ	10 km	1109.25	53
	10 km a 50 km	1021.68	477
	50 km a 110 km	915.56	1131
	110 km a 210 km	744.60	2910
TOTAL OF POINTS			4571

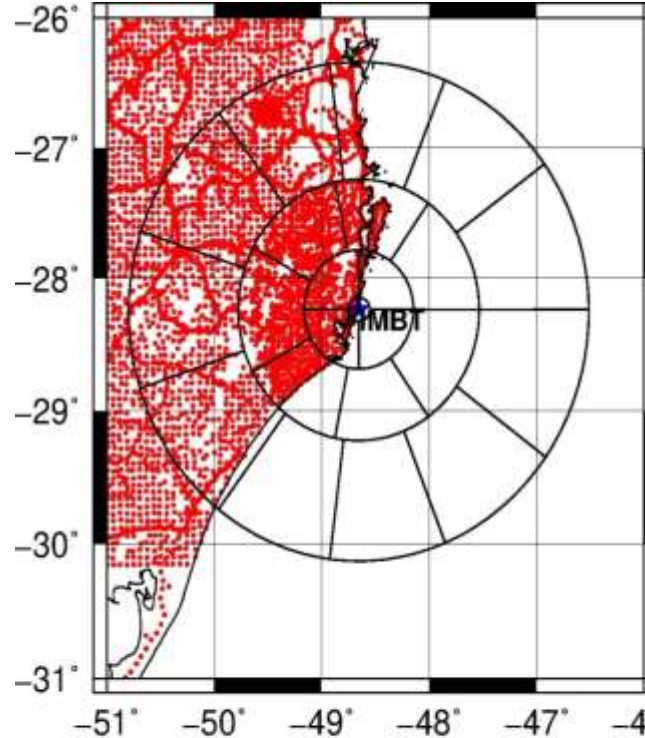
Presidente Prudente (PPTE)



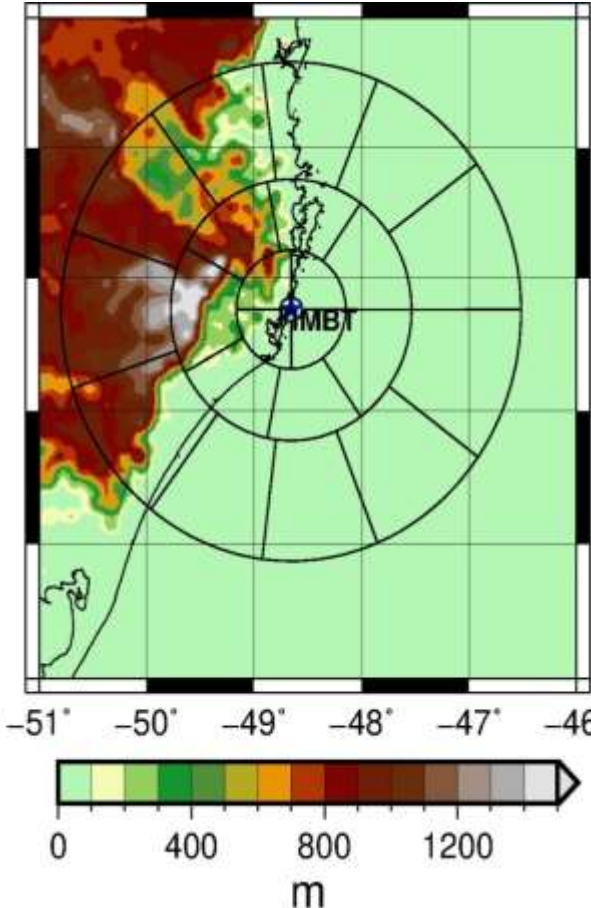
Stations	Distance	H mean	N. of points
PPTE	10 km	445.11	66
	10 km a 50 km	433.26	428
	50 km a 110 km	407.81	1471
	110 km a 210 km	440.45	4101
TOTAL OF POINTS			6066

Imbituba (IMBT)

GRAVITY DISTRIBUTION



TOPOGRAPHY



Stations	Distance	H mean	N. of points
IMBT	10 km	16.50	51
	10 km a 50 km	132.32	311
	50 km a 110 km	392.00	1228
	110 km a 210 km	581.91	2377
TOTAL OF POINTS			3967

Continental Area

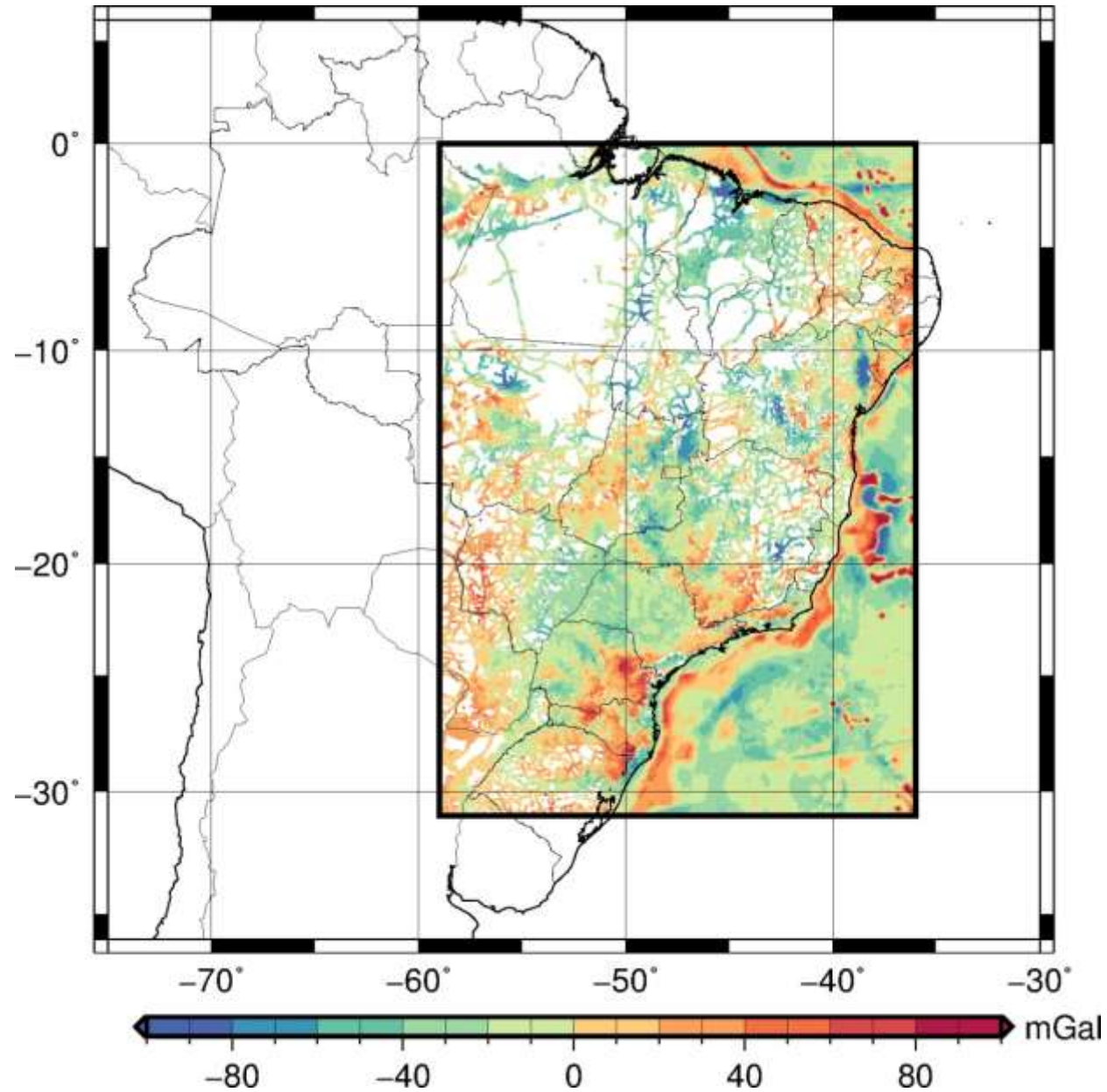
XGM16 (Pail et al., 2016): to obtain elipsoidal height in gravity stations where just have orthometric height.

GOCO05s (Torsten Mayer-Guerr and the GOCO Team, 2015): for the long wavelength component in the remove-restore technic.

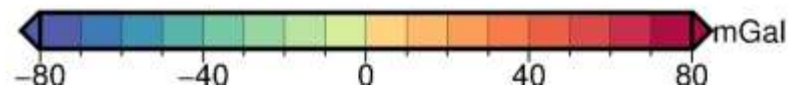
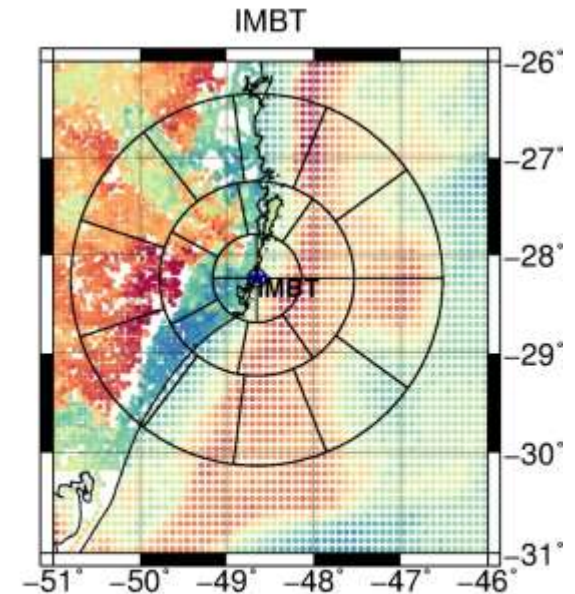
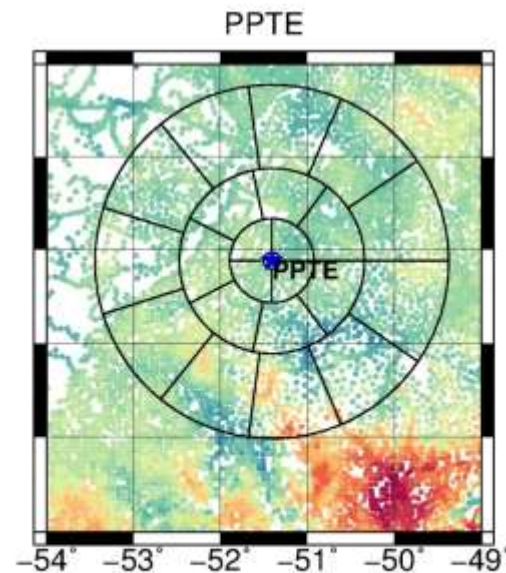
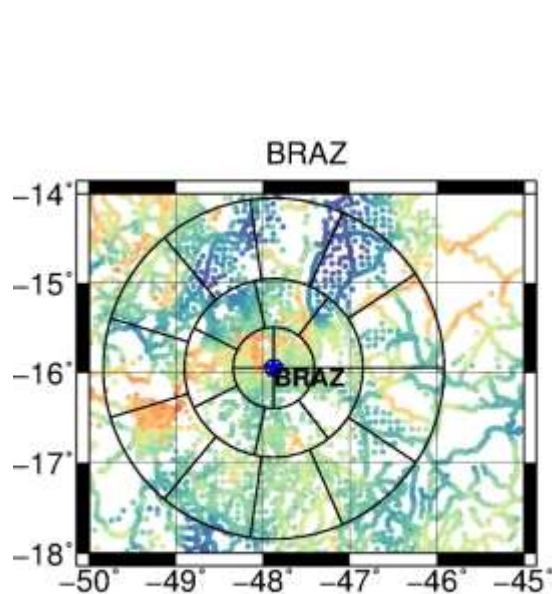
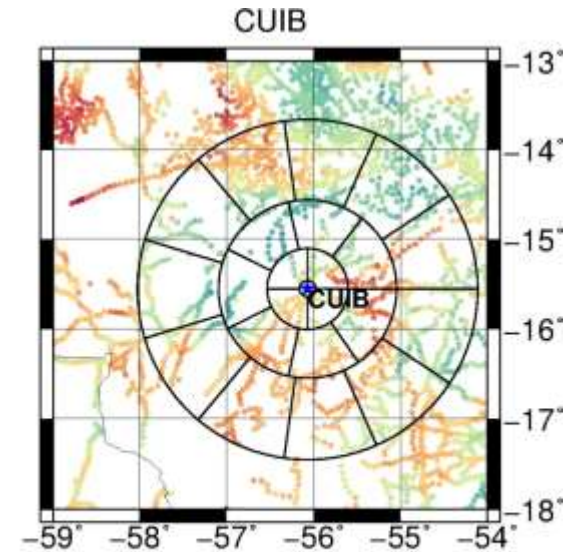
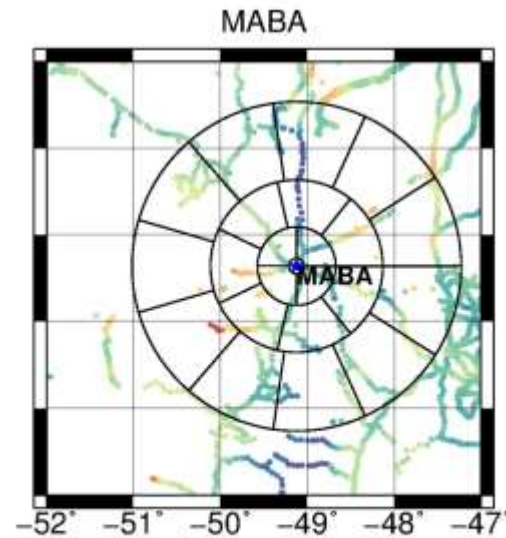
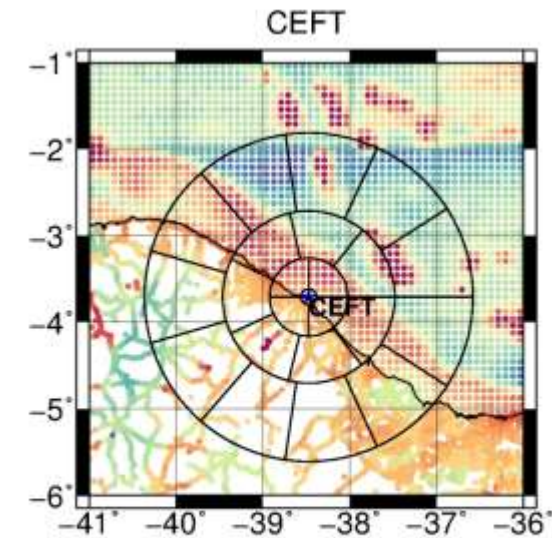
Ocean Area

(Sandwell et al., 2014) – version 24.1: for the marine gravity anomaly from satellite altimetry (1' resolution).

Gravity Disturbance Computation



Gravity Disturbance Computation



Disturbing Potential Computation

$$W_P = U_P + T_P$$



$$T_P = T_{P,satellite-only} + T_{P,residual} + T_{P,terrain}$$



One GGM



GOCO05s (d/o100 and 200)



Terrestrial
gravity
data



One DTM



Not used yet!



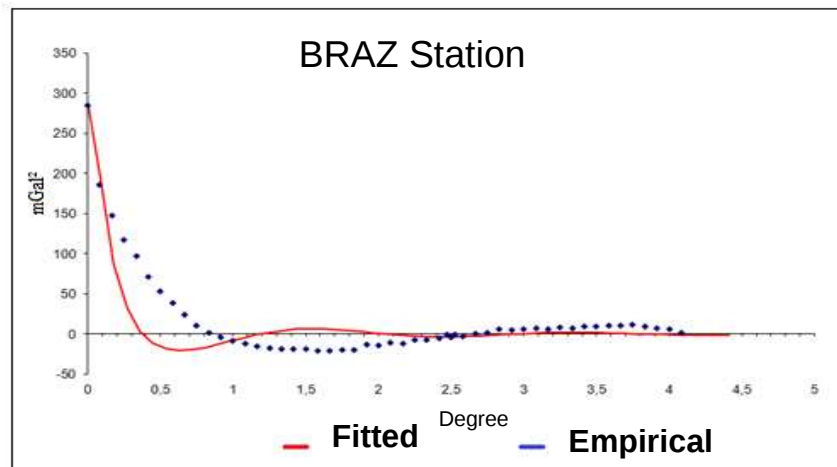
Least Squares Collocation (Fast Collocation)

(BOTTONI; BARZAGHI,1993)

Covariance function is based in :

$$K(P,Q) = \sum_{n=2}^N k_n \left(\frac{R_B^2}{r_P r_Q} \right)^{n+2} P_n(\cos\psi) + \sum_{n=N+1}^{\infty} \frac{A(n-1)}{(n-2)(n+24)} \left(\frac{R_B^2}{r_P r_Q} \right)^{n+2} P_n(\cos\psi)$$

(TSCHERNING; RAPP, 1974)



Hotine's Integral (Numerical Integration)

$$T(P) = \frac{R}{4\pi} \iint \delta g H(\psi) d\sigma$$

$$H(\psi) = \sum_{n=0}^{\infty} \frac{2n+1}{n+1} P_n(\cos\psi)$$

Hotine-Koch function:

$$H(\psi) = \frac{1}{\sin(\psi/2)} - \ln \left(1 + \frac{1}{\sin(\psi/2)} \right)$$

Results – Disturbing Potential (m²/s²)

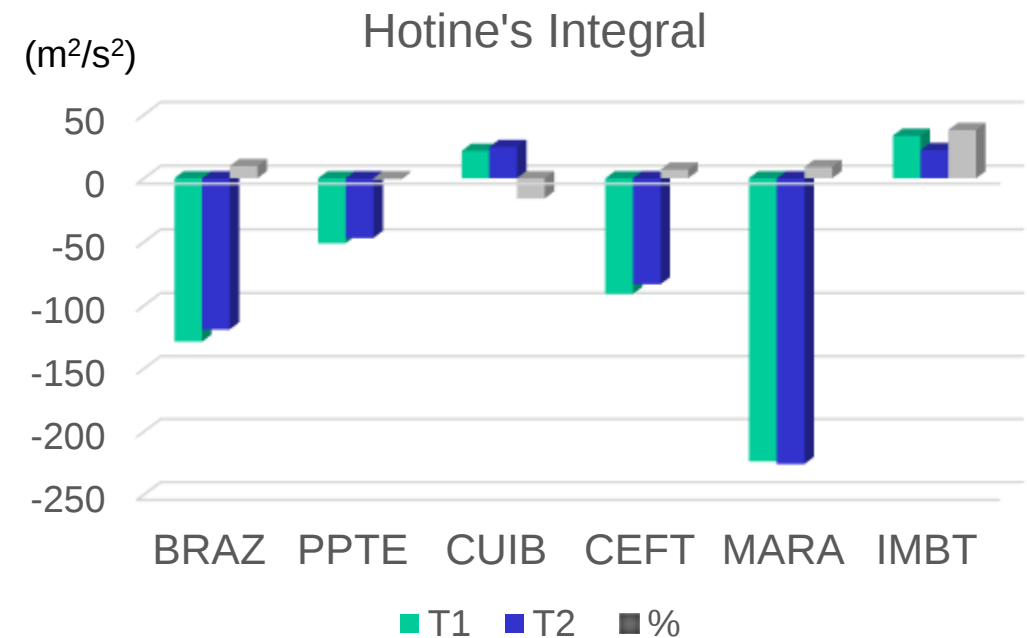
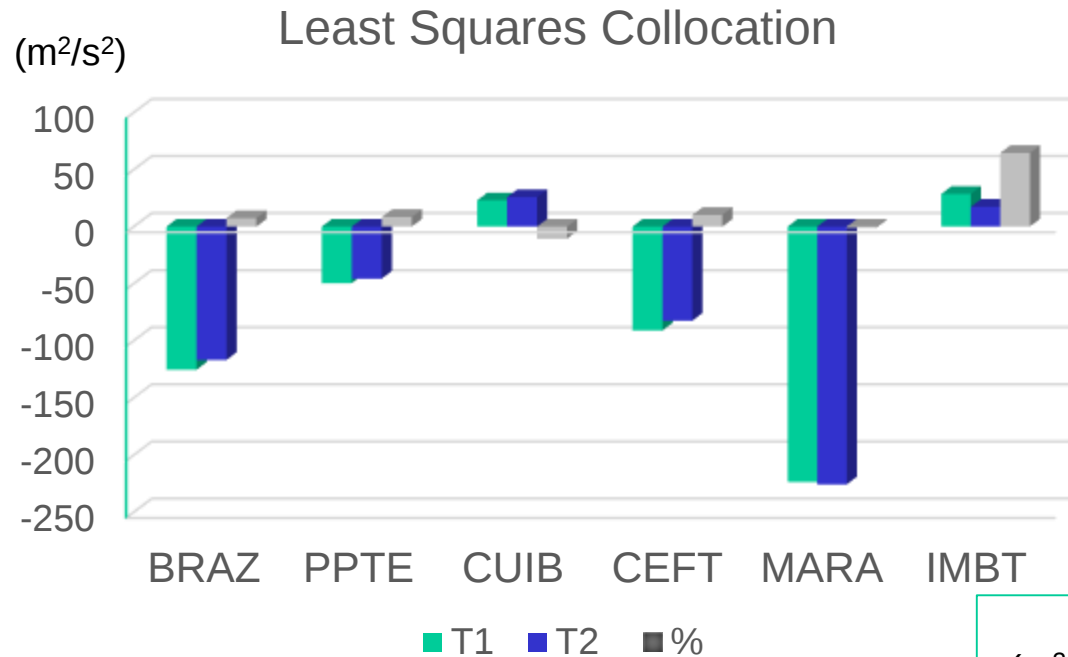
T 1 (d/o 100)

Stations	Hotine's Integral	LSC	Difference
BRAZ	-128.671	-125.500	-3.172
PPTE	-50.760	-49.565	-1.195
CUIB	21.508	23.014	-1.505
CEFT	-90.946	-90.984	0.038
MARA	-223.630	-223.738	0.108
IMBT	33.439	28.518	4.920

T 2 (d/o 200)

Stations	Hotine's Integral	LSC	Difference
BRAZ	-119.041	-116.847	-2.194
PPTE	-46.693	-45.744	-0.949
CUIB	24.506	25.757	-1.251
CEFT	-83.136	-82.544	-0.592
MARA	-225.777	-225.850	0.072
IMBT	22.094	17.327	4.767

Results – Disturbing Potential



Difference between T1 and T2 (m²/s²)

	Hotine's Integral	LSC
Stations	T1-T2	T1-T2
BRAZ	-9.630	-8.653
PPTE	-4.067	-3.821
CUIB	-2.997	-2.743
CEFT	-7.810	-8.440
MARA	2.147	2.112
IMBT	11.344	11.191

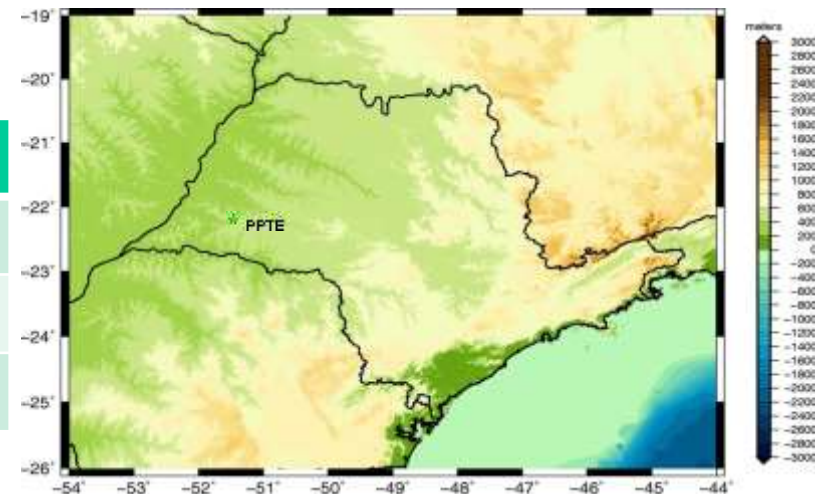
Results – Disturbing Potential

Using Residual Terrain Model (RTM) at PPTE station

	d/o 100 (m ² /s ²)		
	Hotine's Integral	LSC	Difference
No RTM	-50.760	-49.565	-1.195
RTM	-50.507	-49.733	-0.774

	d/o 200 (m ² /s ²)		
	Hotine's Integral	LSC	Difference
No RTM	-46.693	-45.744	-0.949
RTM	-46.734	-45.681	-1.053

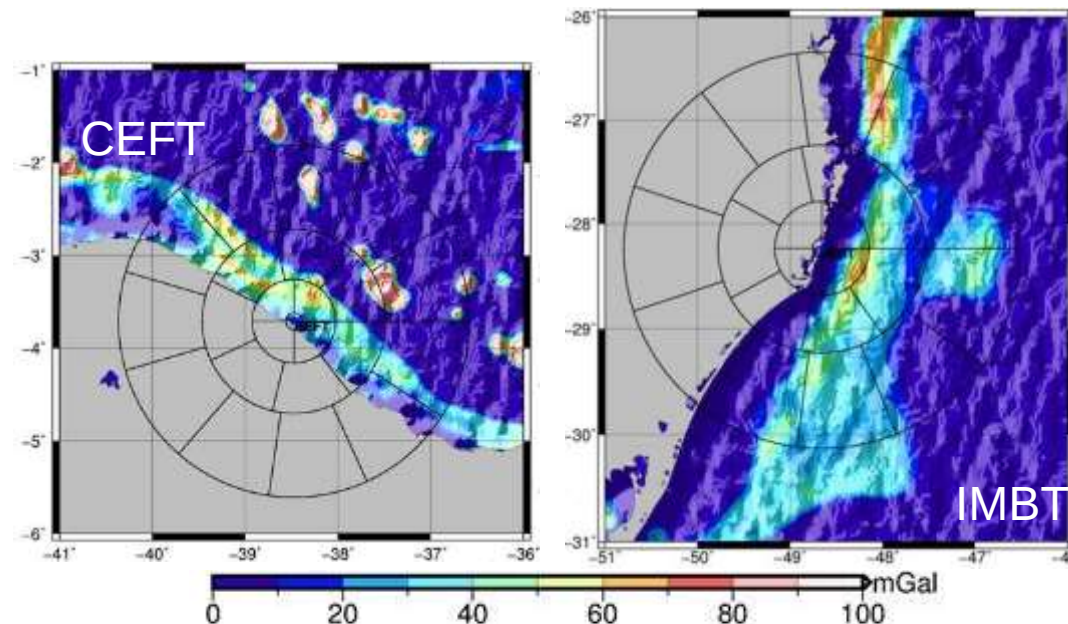
↓
Valéria's talk!



SAM_3sv2 DTM model

Remarks and Next Steps

- ❑ The main idea of this study was to analysis gravity data distribution (radius 210 km), also to evaluate the geopotential model representativity (degree and order) around the 6 IHRF stations. **It is a first step for a long walk!**
- ❑ Two methodologies were used: Hotine's integral and Least Squares Collocation. The results show consistency between them, except IMBT and BRAZ stations.
- ❑ IMBT and CEFT stations are located close to the coast. They are influenced by the satellite altimetry model.



Gravity anomalies errors estimation for
Sandwell et al. (2014) model

☐ Next Steps:

- ☐ To measure terrestrial gravity data in MARA and CUIB stations.
- ☐ To measure absolute gravity in IMBT, MARA and CEFT stations.
- ☐ To compute refined solutions (and to include a DTM model in all stations) and to use others GGMs.