Global vertical datum unification based on the combination of the fixed gravimetric and the scalar free geodetic boundary value problems

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contribution to the IAG-ICP1.2: Vertical Reference Frames

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Motivation

World Height System (WHS)
(IAG-ICP1.2: Vertical Reference Frames, Ihde et al. 2007)

Consistent modelling of geometric and physical parameters, i.e.
\[ h = H^N + \zeta (\approx H + N) \] in a global frame with high accuracy (> \(10^{-9}\))

Geometrical Component

Coordinates:
\[ h(t), \frac{dh}{dt} \]

Definition:
ITRS + Level ellipsoid \( (h_0 = 0) \)
- \((a, J2, \omega, GM)\) or
- \((W_0, J2, \omega, GM)\)

Realization:
1. Related to the ITRS (ITRF)
2. Conventional ellipsoid

Conventions:
IERS Conventions

Ellipsoid constants, \(W_0, U_0\) values, reference tide system have to be aligned to the physical conventions!

Physical Component

Coord.: Potential differences
\[-\Delta W_p(t) = W_0(t) - W_p(t); d\Delta W_0/dt\]

Definition:
\[ W_0 = \text{const.} \] (as a convention)

Realization:
1. Selection of a global \(W_0\) value
2. Determination of the local \(W_{0,j}\) values
3. Connection of \(W_{0,j}\) with \(W_0\)
4. Geometrical representation of \(W_0\) and \(W_{0,j}\) (i.e. geoid comp.)
5. Potential differences into physical heights \((H\text{ or }H^N)\)

Zero tide system
Considerations on $W_0$, $W_{0,j}$

- The reference level $(W_0, W_{0,j})$ for potential differences can **arbitrarily be appointed**, but it is preferred that this level refers to the mean sea level and it shall be derived from **actual observations** of the Earth’s gravity field and of the sea surface (Gauss/Listing geoid definition);

- The **direct determination** of absolute potential values $(W_0, W_{0,j})$ from observational data is **not possible**, adequate **constraints** are required;

- These constraints (mainly the vanishing of the gravitational potential $V$ at infinity) are **only reliable** in the frame of the **Geodetic Boundary Value Problem** (GBVP); hence, the determination of suitable $W_0$ or $W_{0,j}$ values is exclusively feasible by solving the GBVP;

- This procedure reduces the ‘arbitrariness’ of the reference level; the **obtained $W$ values** will be in **agreement** with the **geodetic observations** included for solving the GBVP.
### Determination of $W_0$ and $W_{0,j}$

**Ocean areas**

<table>
<thead>
<tr>
<th>Fixed gravimetric GBVP</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Estimation of the potential of the <strong>level surface</strong> that best approximates the <strong>mean sea surface</strong></td>
</tr>
<tr>
<td>✓ Geometry of the boundary surface (mean sea surface) is known from <strong>satellite altimetry</strong></td>
</tr>
<tr>
<td>✓ This value is appointed as the global reference level $W_0$</td>
</tr>
</tbody>
</table>

**Land areas**

<table>
<thead>
<tr>
<th>Scalar-free GBVP (Molodensky App)</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Since the observational data included in the boundary conditions <strong>refer to different vertical datums</strong>, we obtain as many $W_{0,j}$ values as existing height systems $j$.</td>
</tr>
<tr>
<td>✓ GVBP shall <strong>homogeneously</strong> be solved in all datum zones ($j = 1 \ldots J$); i.e, gravity anomalies at ground level, the same GGM, the same reference ellipsoid, etc.</td>
</tr>
</tbody>
</table>

Relationships $W_0 - W_{0,j}$, $W_{0,j} - W_{0,j+1}$ through vertical datum unification strategies
### W₀ (W₀,j) in the GBVP frame

<table>
<thead>
<tr>
<th>Ocean areas</th>
<th>Land areas</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formulation</strong></td>
<td><strong>Constraints</strong></td>
</tr>
<tr>
<td>( \nabla^2 T = 0 ) outside boundary surface ; ( T = W - U )</td>
<td>( - \frac{\partial T}{\partial r} = \delta g )</td>
</tr>
<tr>
<td>( \frac{\partial T}{\partial r} - \frac{2}{r} T = g_j - \frac{2}{r} \delta W_j )</td>
<td>( \delta W_j = W_0,j - U_0 = W_0 - W_0,j )</td>
</tr>
<tr>
<td><strong>Solution</strong></td>
<td><strong>Solution</strong></td>
</tr>
<tr>
<td>( T = \frac{\Delta GM}{R} + \frac{R}{4\pi} \int \int B_j S(\psi) d\sigma + \frac{R}{4\pi} \int \int G_n S(\psi) d\sigma )</td>
<td>( j = 1 \ldots J ; \quad B_j = g_j - \frac{2}{r} \delta W_j )</td>
</tr>
<tr>
<td>( j = 1 ; B_1 = \delta g ) (gravity disturbances)</td>
<td>( g_1 = \Delta g ; \ g_2 = \Delta C ; \ etc. )</td>
</tr>
<tr>
<td>S(\psi) = S'(\psi) = \frac{1}{\sin(\psi/2)} - \ln\left(1 + \frac{1}{\sin(\psi/2)}\right)</td>
<td>S(\psi) = \frac{1}{\sin(\psi/2)} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi ...</td>
</tr>
<tr>
<td><strong>Results</strong></td>
<td><strong>Results</strong></td>
</tr>
<tr>
<td>( W_p = U_0 - \gamma_p h_p + T_p )</td>
<td>( \zeta_j = \frac{T + \delta W_j}{\gamma} )</td>
</tr>
<tr>
<td>( W_0 = \int \frac{W_p}{\gamma_p^2} d\sigma \int \frac{1}{\gamma_p^2} d\sigma )</td>
<td></td>
</tr>
</tbody>
</table>
Numerical results: $W_0$ value

Solution of the fixed gravimetric GBVP taken as input data:

Geometry of the mean sea surface: **CLS01** model (Hernandez, Schaeffer 2001),
**DGFI annual models** derived from T/P

Gravity disturbances from GGM: **EGM2008** model (Pavlis et al. 2008) and
**EIGEN-GL04S** (GRGS/GFZ 2006)

Other $W_0$ computations:

Best fitting ellipsoid:

$U_0 = 62 636 \, 860,850 \, m^2 s^{-2}$ (GRS80)
$\quad 856,88$ (Rapp, 1995)

Mean potential value from

$$ W = \frac{GM}{r} \left[ 1 + \sum_{n=1}^{\infty} \sum_{m=0}^{n} C_{nm} \cos m\lambda \right] $$

$W_0 = 62 636 \, 857,5$ (Nesvorny and Sima 1994)
$\quad 856,5$ (Ries 1995)
$\quad 856,0$ (Bursa et al. 2002)
$\quad 854,7$ (Bursa et al. 2006)
$\quad 853,4$ (Sánchez 2005)
Constraint for the empirical determination of the $\delta W_j$ terms: $\gamma_p h_p - (W_{0j} - W_{pj}) - T_{pj} - 2\delta W_j = 0$
Observation equations for Vertical datum unification

<table>
<thead>
<tr>
<th>Approach</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Oceanic approach</strong></td>
<td>(SSTop around tide gauges)</td>
<td>$T_p^j - T_0 = \delta W^j$</td>
</tr>
<tr>
<td></td>
<td>Data: Satellite altimetry and satellite-only GGM, SSTop at coast lines by including also tide gauge records.</td>
<td></td>
</tr>
<tr>
<td><strong>Coastal approach</strong></td>
<td>(reference tide gauges)</td>
<td>$\frac{1}{2} T_p^j - \frac{1}{2} h_p \gamma_p = \delta W^j$</td>
</tr>
<tr>
<td></td>
<td>Data: GPS positioning at tide gauges, spirit levelling with gravity corrections, terrestrial gravity data and satellite-only GGM.</td>
<td></td>
</tr>
<tr>
<td><strong>Continental approach</strong></td>
<td>(geometric reference stations)</td>
<td>$\frac{1}{2} \left( W_0^j - W_p^j + T_p^j \right) - \frac{1}{2} h_p \gamma_p = \delta W^j$</td>
</tr>
<tr>
<td></td>
<td>Data: GPS positioning at reference stations (including border points), spirit levelling with gravity corrections, terrestrial gravity data and satellite-only GGM.</td>
<td>$\frac{1}{2} \left( W_0^j - W_p^j + T_p^j \right) - \frac{1}{2} \left( W_0^{j+1} - W_p^{j+1} + T_p^{j+1} \right) = \delta W^{j+1} - \delta W^j$</td>
</tr>
</tbody>
</table>
Numerical results: SIRGAS example

Input data:
Local quasigeoid models
GNSS positioning, mean sea surface heights,
Geopotential numbers from levelling
$H^N$, $h$, $\zeta$, SSTop at epoch 2000.0, zero tide system
Numerical results: SIRGAS example

**ECUADOR**
- Tide gauge
- SIRGAS2000 station
- Levelling point
Closing remarks

- The determination of $\delta W_i$ must be based on regional geoids of high resolution. The GGMs do not provide the required accuracy and resolution.

- $\delta W_i$ terms shall be estimated at the definition period of the local reference levels, i.e. the sea level rise and the vertical crustal movements must be taken into account, and all heights ($h$, $H^N$, $\zeta$, $\text{SSTop}$) shall be reduced to a reference epoch.

- The determination of $\delta W_i$ requires the three proposed approaches: coastal, terrestrial, and oceanic approach. Their isolated evaluation leads to unreliable values.

- The discussion about introducing orthometric or normal heights should be a question of the realization, not of the definition. However, the global vertical system must support both types of heights. In this way, its reference level should be determined where both surfaces (geoid and quasigeoid) are the same: in oceanic areas.

- Although the reference level should be defined by a fixed $W_0$ value (for the computation of geopotential numbers), it must also be realized geometrically by the (quasi)geoid determination (solution of the GBVP).

- The uniqueness, reliability and repeatability of the global reference level $W_0$ can be guaranteed by introducing specific conventions only, e.g. $V_\infty=0$, mean sea surface model, global gravity model, tide system, reference epoch, etc. On the contrary, it will be exist as many height systems as $W_0$ computations.