The International Height Reference System (IHRS) and its realisation, the International Height Reference Frame (IHRF)

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Workshop for the Implementation of the GGRF in Latin America
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Outline

- Motivation
- Definition of the International Height Reference System (IHRS)
- Realisation of the IHRS: the International Height Reference Frame (IHRF)
    - Station selection for the reference network
    - Some considerations for the determination of IHRS/IHRF coordinates
- Colorado experiment: comparison of potential values and learnings from a successful international cooperation initiative
- Participation of Latin America in the implementation of the IHRS/IHRF

Motivation

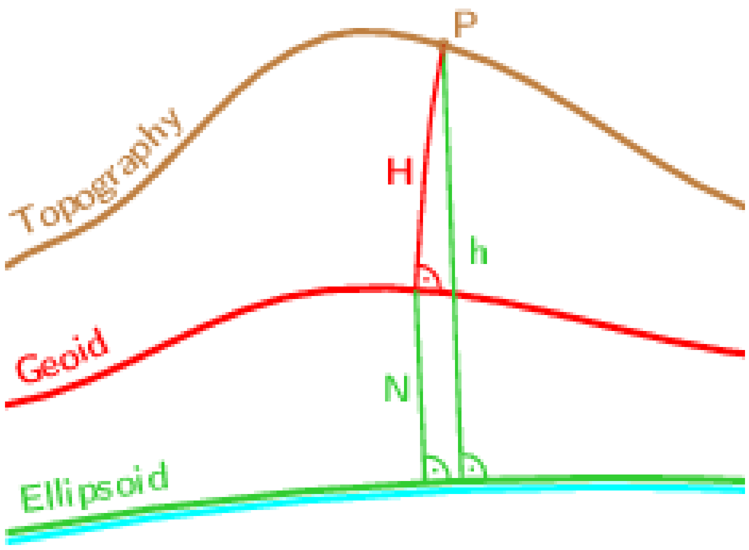
1) Vertical coordinates used in practice:

- h → ellipsoidal heights (GNSS positioning);
- H → Physical heights (levelling + gravity reductions);
- N → (Quasi-)geoid undulations (gravity field modelling).

2) Everyone using GNSS positioning and requiring physical heights demands

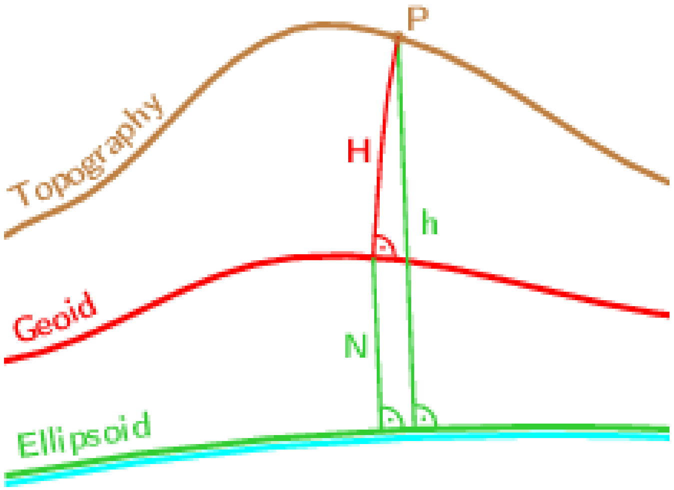
$$H = h - N$$

with consistency at the cm-level and worldwide.

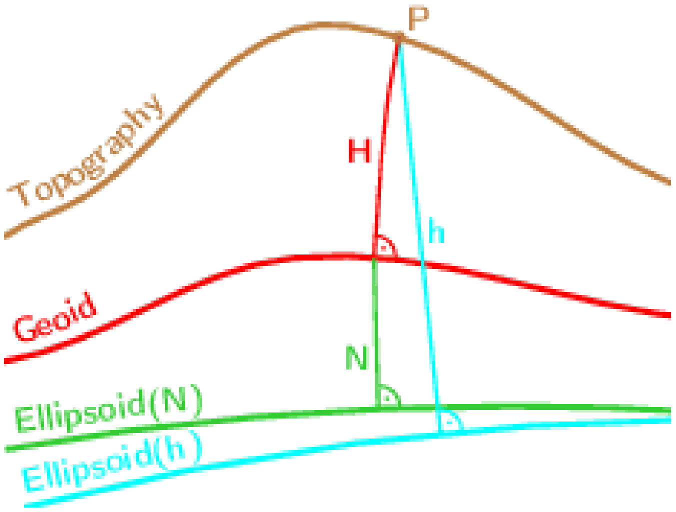


H = h - N in theory ... but in practice, e.g.

- Ellipsoidal heights h and geoid undulations N must be given w.r.t. the same ellipsoid:
  - [X, Y, Z] ⇔ [φ, λ, h]
  - Reference field (surface) for the geoid computation and for scaling global gravity models (GGM)

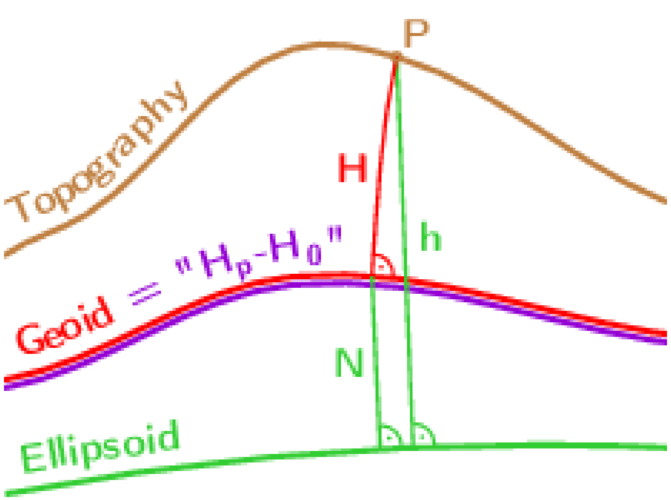


- Different ellipsoid parameters (a, GM) are used in geometry and gravity, for instance:
  - Geometric coordinates [φ, λ, h] referring to the GRS80 ellipsoid or to the WGS84 ellipsoid are practically identical
  - Geoid undulations N referring to the WGS84 ellipsoid present a discrepancy of about 93 cm w.r.t. geoid undulations referring to the GRS80



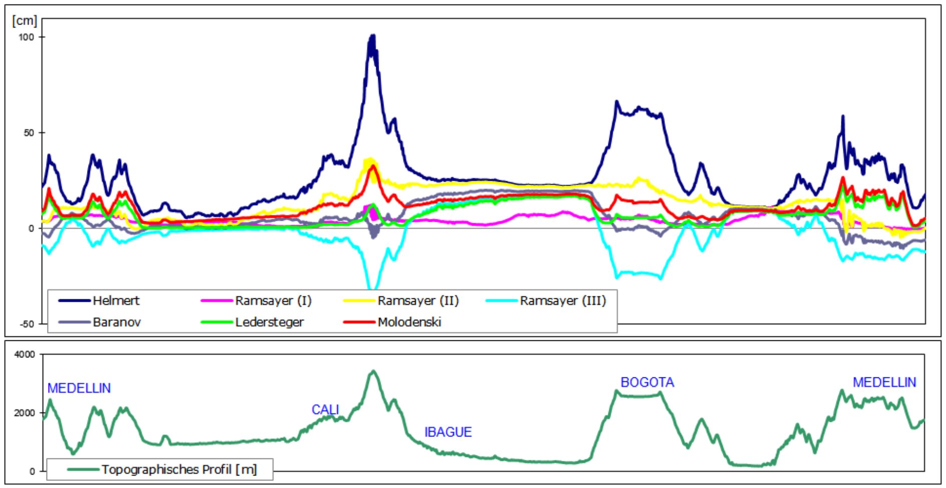
$H = h - N$ in theory ... but in practice, e.g.

- Physical heights H and (quasi)geoid undulations N must reflect the same reference surface:
  - H_p (from levelling) – H_0 (datum point)
    $\rightarrow$ geoid from geometry
  - N (from the GBVP)
    $\rightarrow$ geoid from gravity



- Orthometric heights and gravimetric geoid use different hypotheses
- Different tide systems for H and N
- Systematic errors over long distances in levelling (reliability of H_p-H_0)

Levelling-based physical heights with different gravity corrections



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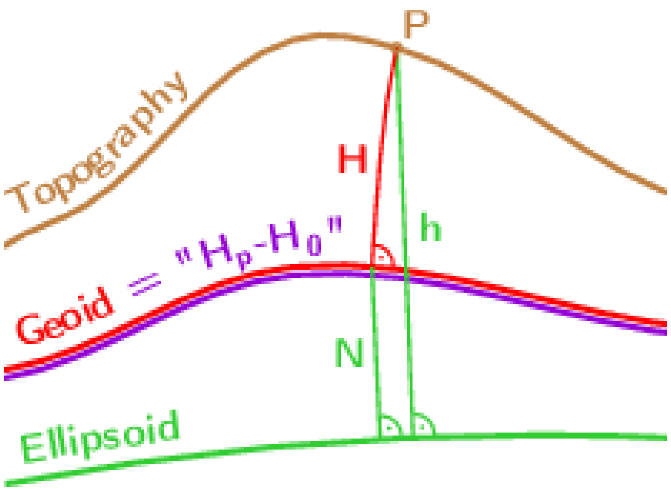

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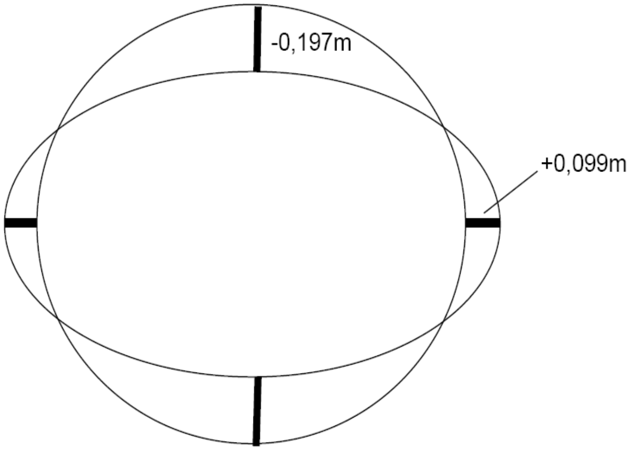
Differences between mean and zero geoids (Heck 2010)



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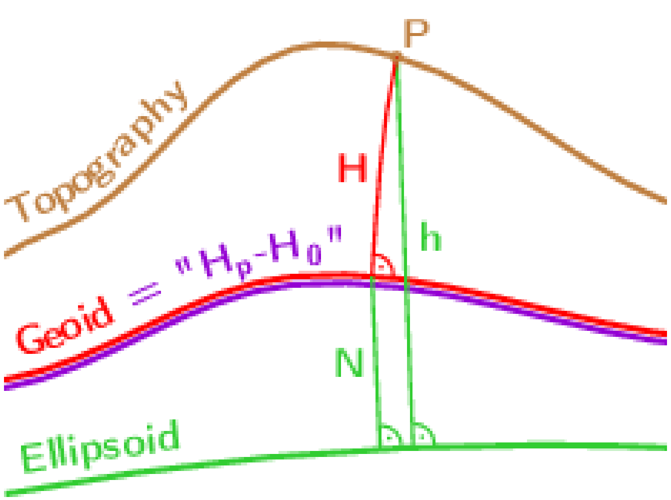

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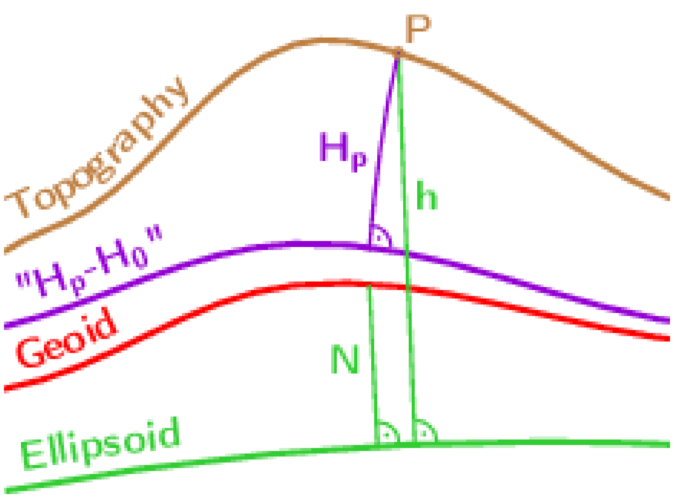
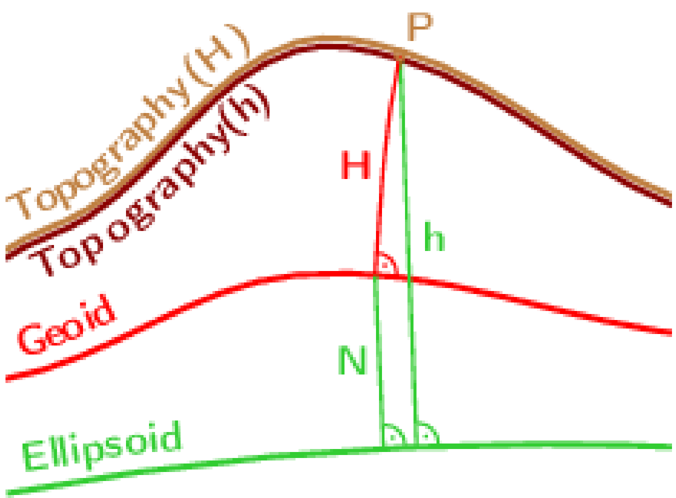


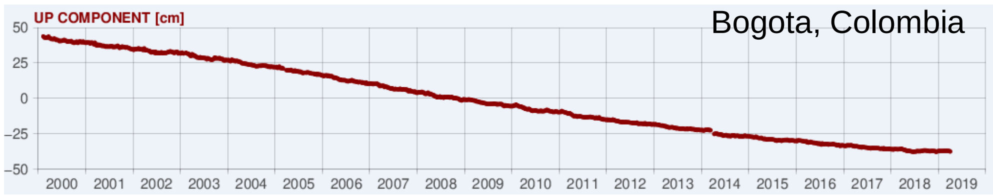
H = h - N in theory … but in practice, e.g.

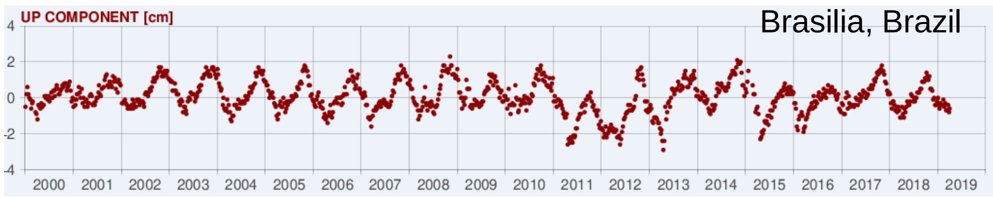
- Physical heights H and ellipsoidal heights h must represent the same Earth’s surface



- Different reference epochs (with unknown dH/dt)
- Different reductions (Earth-, ocean-, atmospheric tides, ocean and atmospheric loading, post-glacial rebound, etc.)

Time series of ellipsoidal heights,

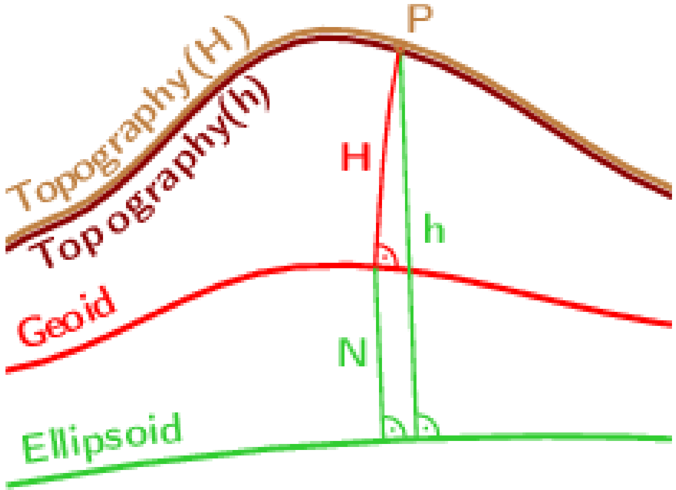




but levelling-based physical heights constant (dH/dt = 0)

H = h - N in theory ... but in practice, e.g.

- Physical heights H and ellipsoidal heights h must represent the same Earth's surface



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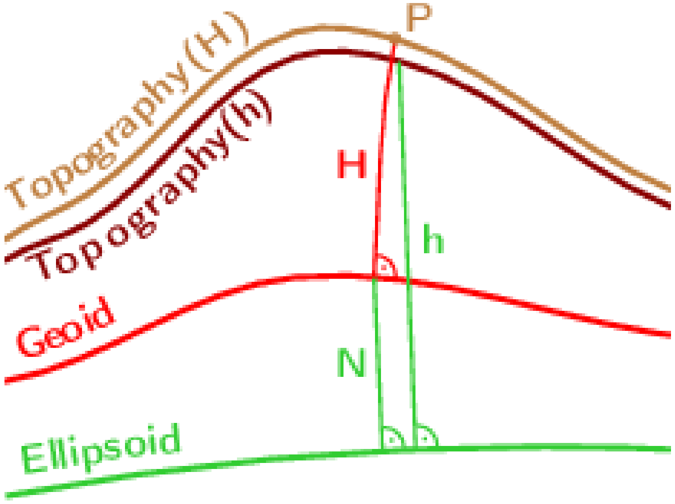
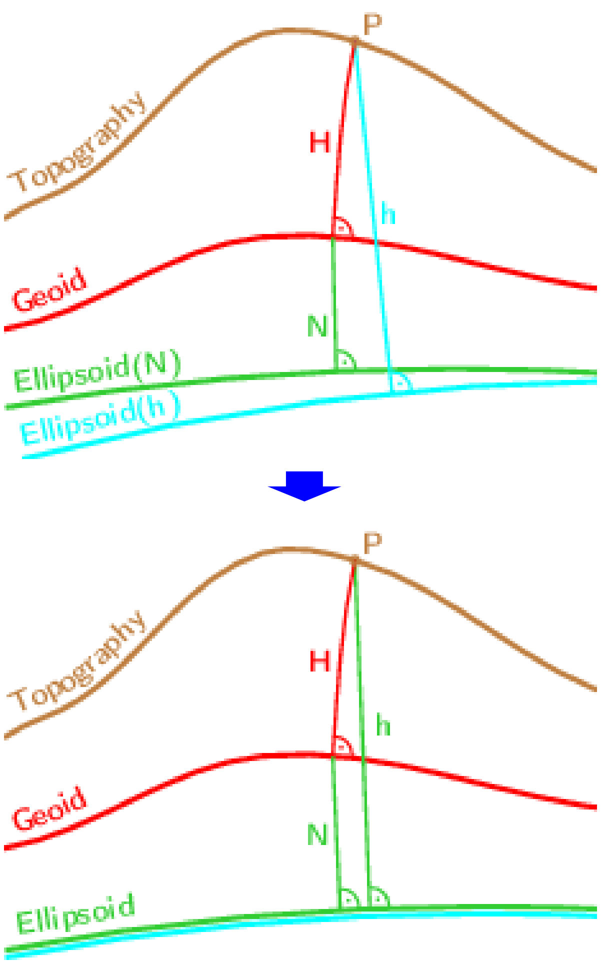
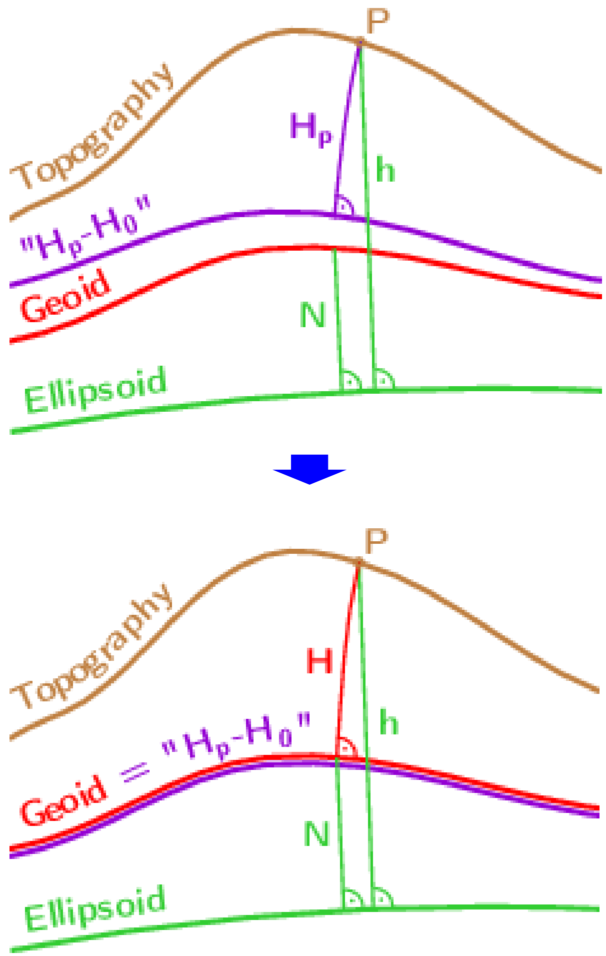


A global unified height system is needed to ensure consistency between

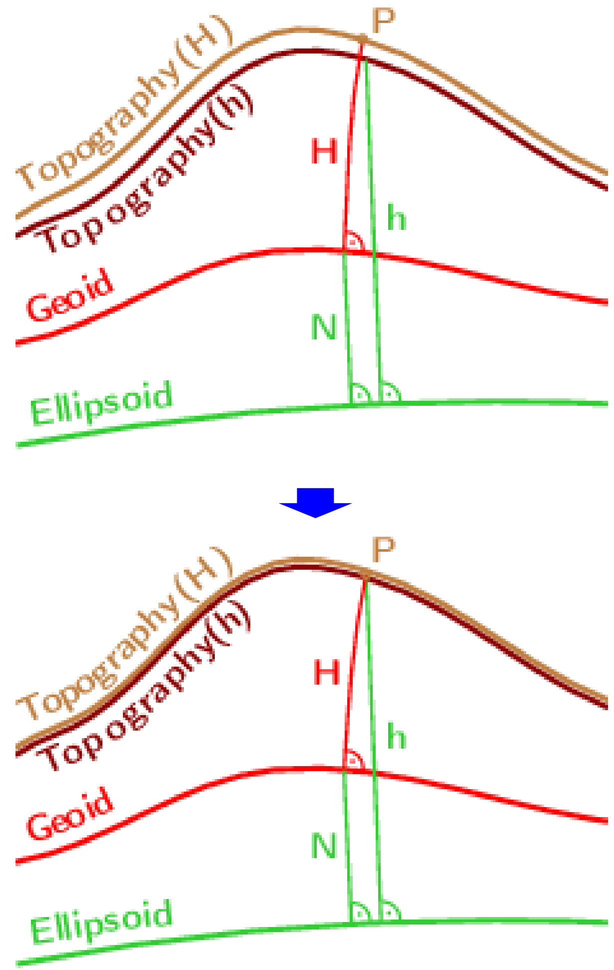
h, N to get H



H, N to get h

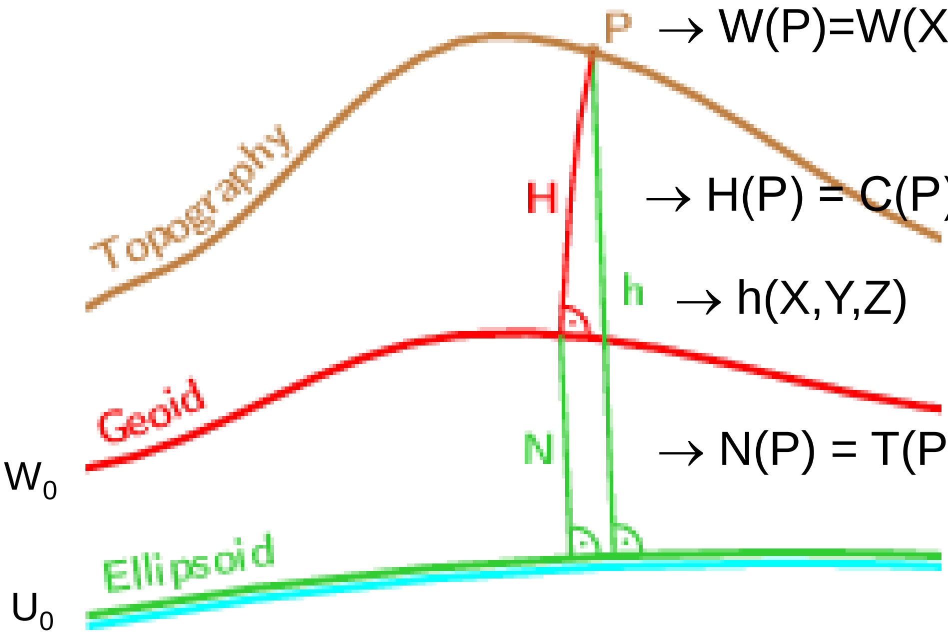


h, H to get N



worldwide and at the cm-level!

Vertical coordinates in terms of potential



$\rightarrow W(P)=W(X,Y,Z)$

$\rightarrow H(P) = C(P)/\hat{g} \rightarrow C(P) = W_0\text{-}W(P)$

$\rightarrow h(X,Y,Z)$

$\rightarrow N(P) = T(P)/\gamma \rightarrow T(P) = W - U$

Definition of the International Height Reference System (IHRS)
IAG Resolution No. 1, Prague, July 2015

1) Vertical coordinates are potential differences with respect to a conventionally fixed W_0 value:

$$C_P = C(P) = W_0 - W(P) = -\Delta W(P)$$

$$W_0 = const. = 62\,636\,853.4 \text{ m}^2\text{s}^{-2}$$

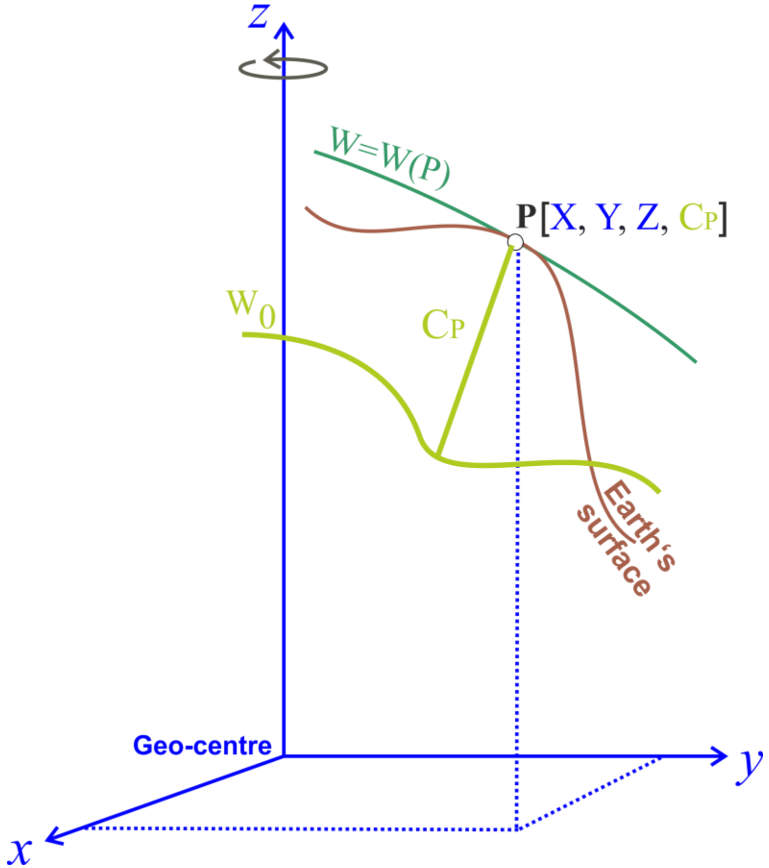
2) The position P is given in the ITRF

$$\mathbf{X}_P\ (X_P, Y_P, Z_P)\text{; i.e., } W(P) = W(\mathbf{X}_P)$$

3) The estimation of $\mathbf{X}(P)$, $W(P)$ (or $C(P)$) includes their variation with time; i.e., $\dot{\mathbf{X}}(P)$, $\dot{W}(P)$ (or $\dot{C}(P)$).

4) Coordinates are given in mean-tide system / mean (zero) crust.

5) The unit of length is the meter and the unit of time is the second (SI).



See: Ihde J. et al.: *Definition and proposed realisation of the International Height Reference System (IHRS)*. Surv Geophy 38(3), 549-570, 10.1007/s10712-017-9409-3, 2017

Realisation of the IHRS

A reference frame realises a reference system in two ways:

- physically, by a solid materialisation of points (or observing instruments),
- mathematically, by the determination of coordinates referring to that reference system.
- The coordinates of the points are computed from the measurements following the definition of the reference system.

During the last four years different actions have been conducted to

- Establish a global reference network for the IHRS realisation: the International Height Reference Frame (IHRF)
- Evaluate different strategies for the determination of reference coordinates at the reference stations
- Identify required standards, conventions and procedures needed to ensure consistency between the definition (IHRS) and the realisation (IHRF).

Criteria for the IHRF reference network configuration

1) Hierarchy:

- A global network $\rightarrow$ worldwide distribution, including
- A core network $\rightarrow$ to ensure sustainability and long term stability
- Regional and national densifications $\rightarrow$ local accessibility

2) Collocated with:

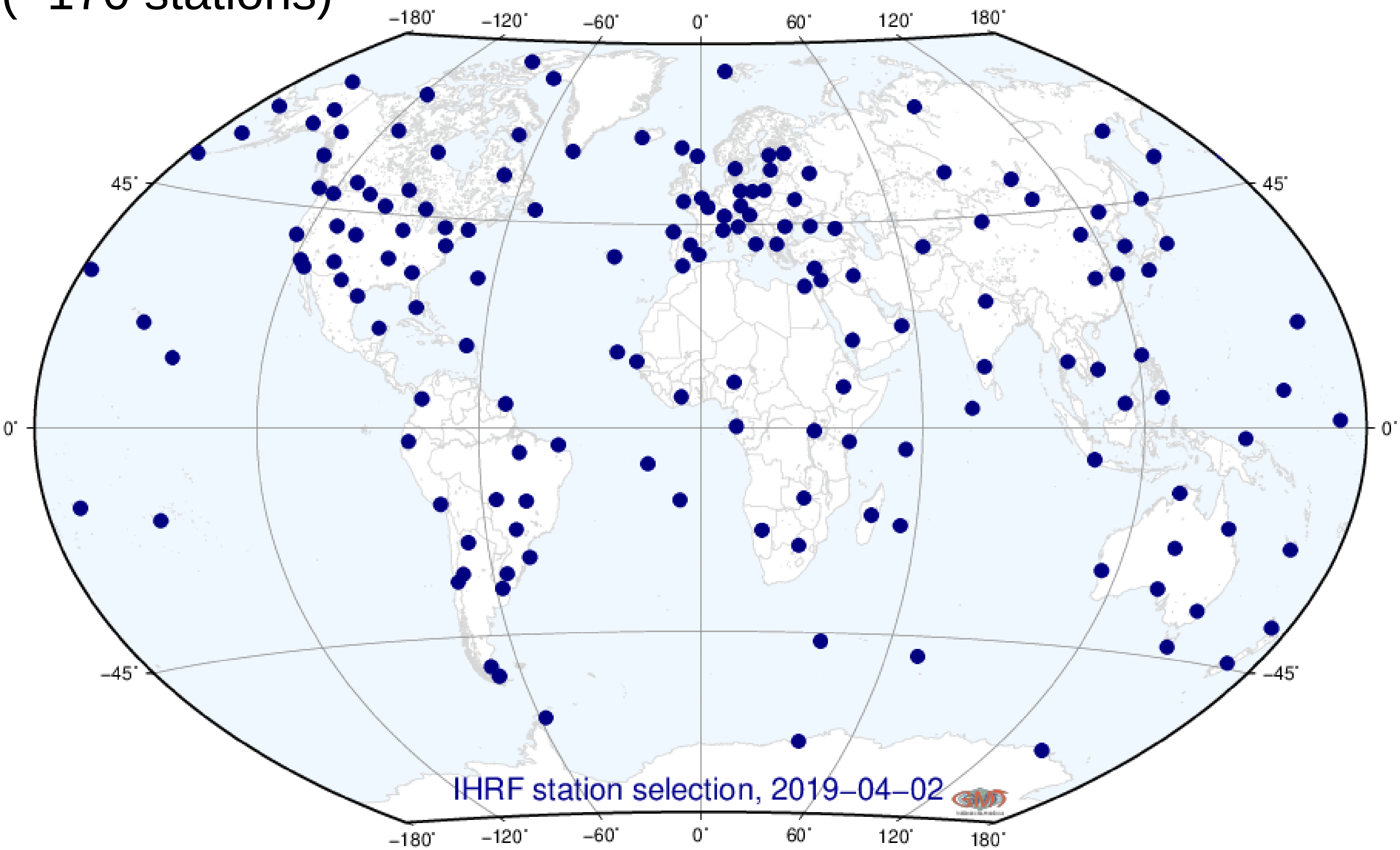
- fundamental geodetic observatories $\rightarrow$ connection between $\mathbf{X}$, W, $\mathbf{g}$ and time realisation (reference clocks) $\rightarrow$ to support the GGRF;
- continuously operating reference stations $\rightarrow$ to detect deformations of the reference frame (preference for ITRF and regional reference stations, like SIRGAS, EPN, APREF, etc.);
- reference tide gauges and national vertical networks $\rightarrow$ to facilitate the vertical datum unification;
- reference stations of the new International Gravity Reference Frame - IGRF (see IAG Resolution 2, Prague 2015).

3) Main requirement: availability of terrestrial gravity data around the IHRS reference stations for high-resolution gravity field modelling (i.e., precise estimation of W).

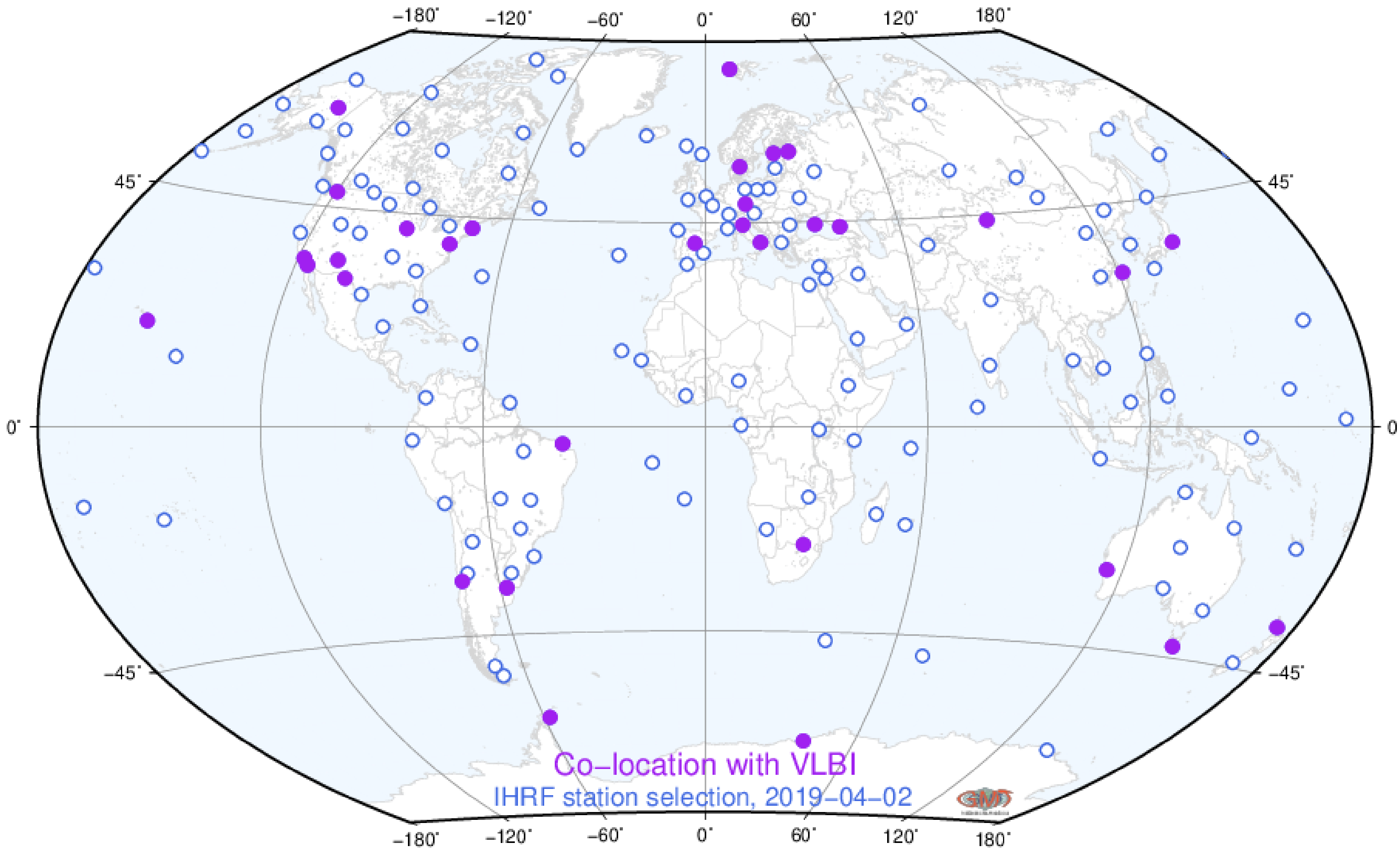
Station selection

1) With the support of the GGOS Bureau for Networks and Observations, a preliminary selection based on VLBI, SLR and DORIS reference sites co-located with GNSS was prepared (Oct 2016).
2) Based on these preliminary selection, national/regional experts were asked to
   - evaluate whether these sites are suitable to be included in the IHRF: Are gravity data around these sites available? If not, is it possible to survey gravity around them?
   - propose additional geodetic sites to improve the density and distribution of the IHRF stations in their regions/countries
3) With support of the IAG JWG 2.1.1: Establishment of a global absolute gravity reference system (chair: H. Wziontek), further stations co-located with absolute gravity stations were identified.
4) A first proposal for the IHRF reference network was ready in Apr 2017.
5) Since that time some new stations have been added, others have been decommissioned.
6) It is expected that this network is extended by means of regional/national densifications.

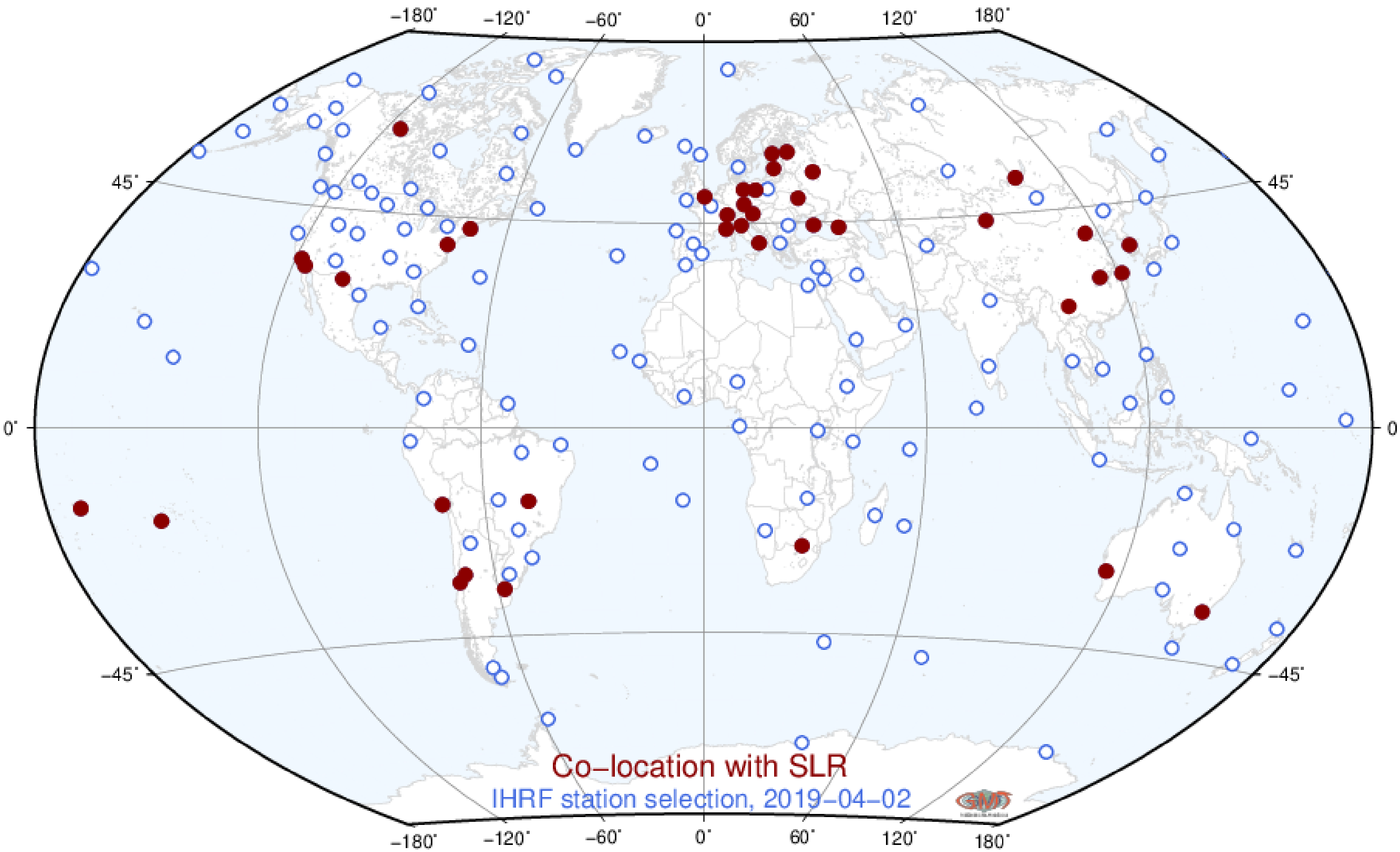
First proposal for the IHRF reference network (~170 stations)



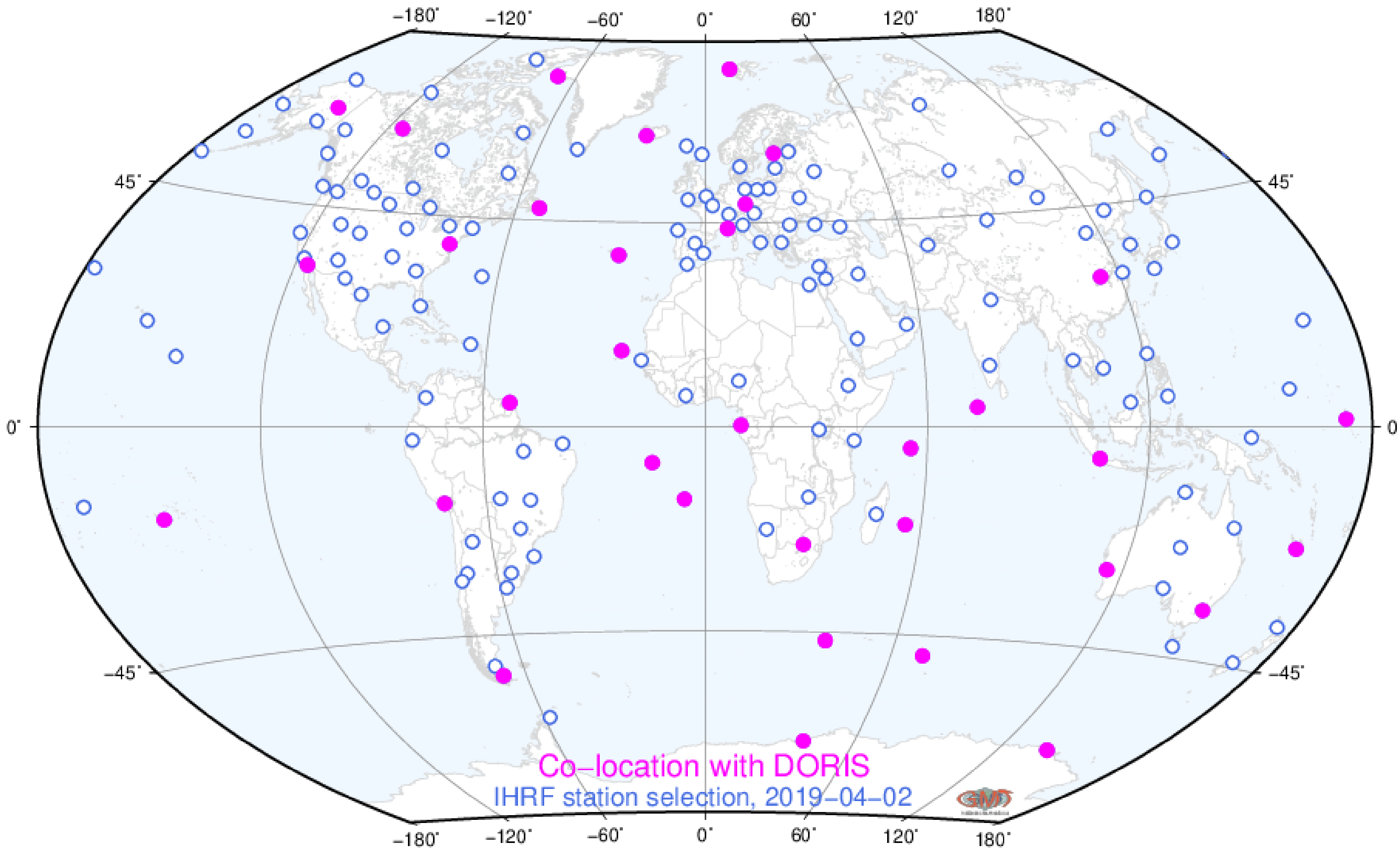
Co-location with VLBI (30 sites)



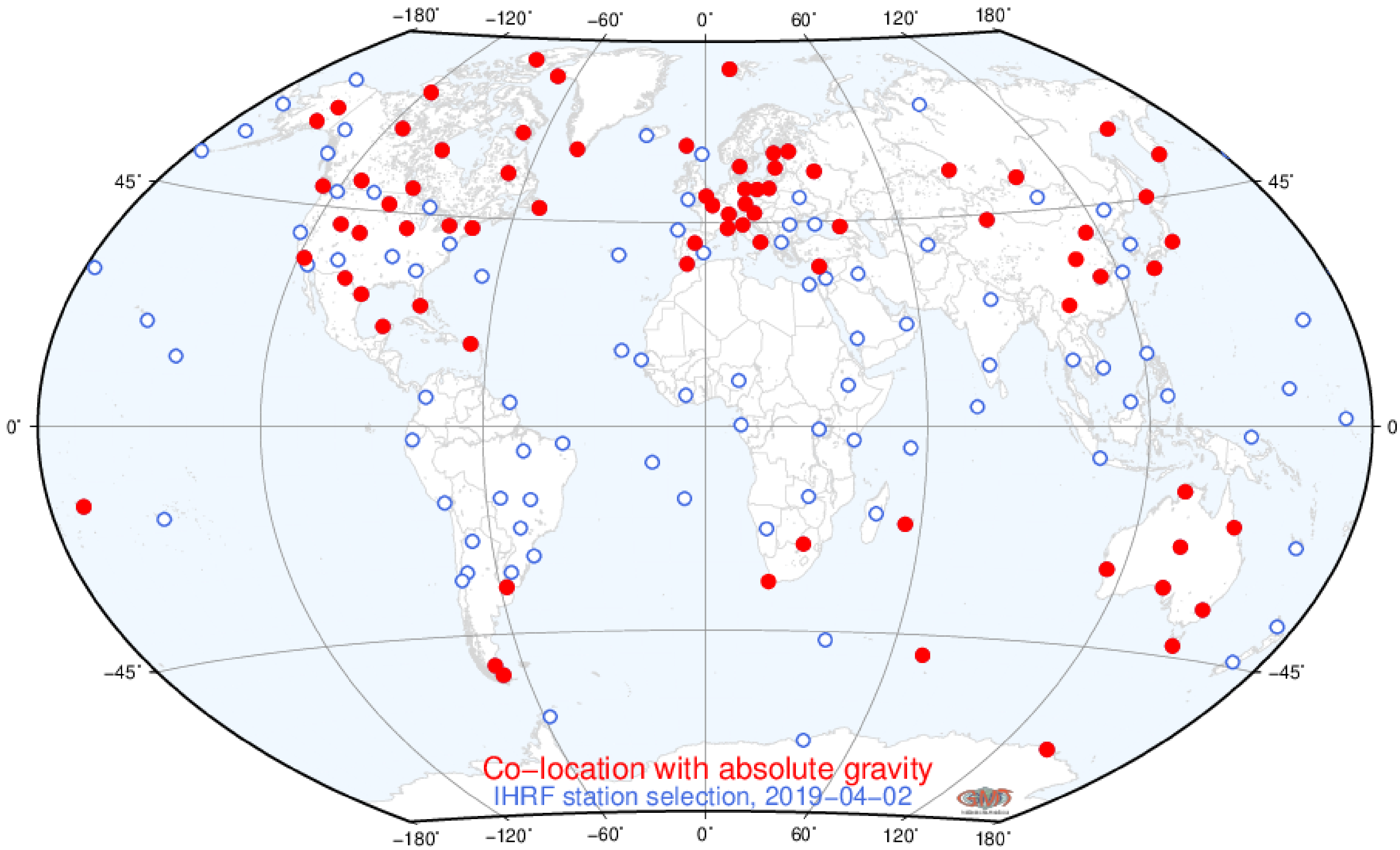
Co-location with SLR (40 sites)



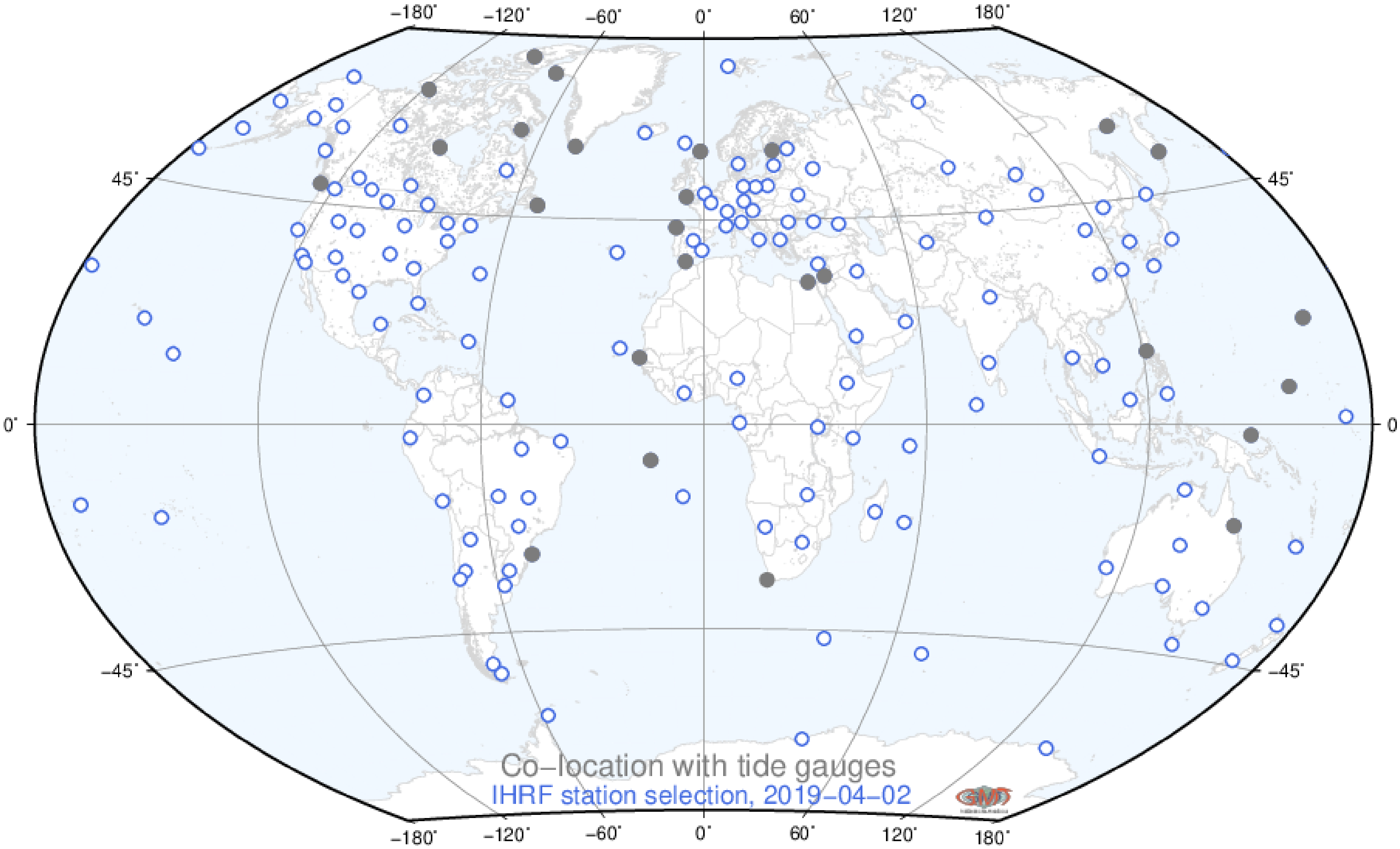
Co-location with DORIS (35 sites)



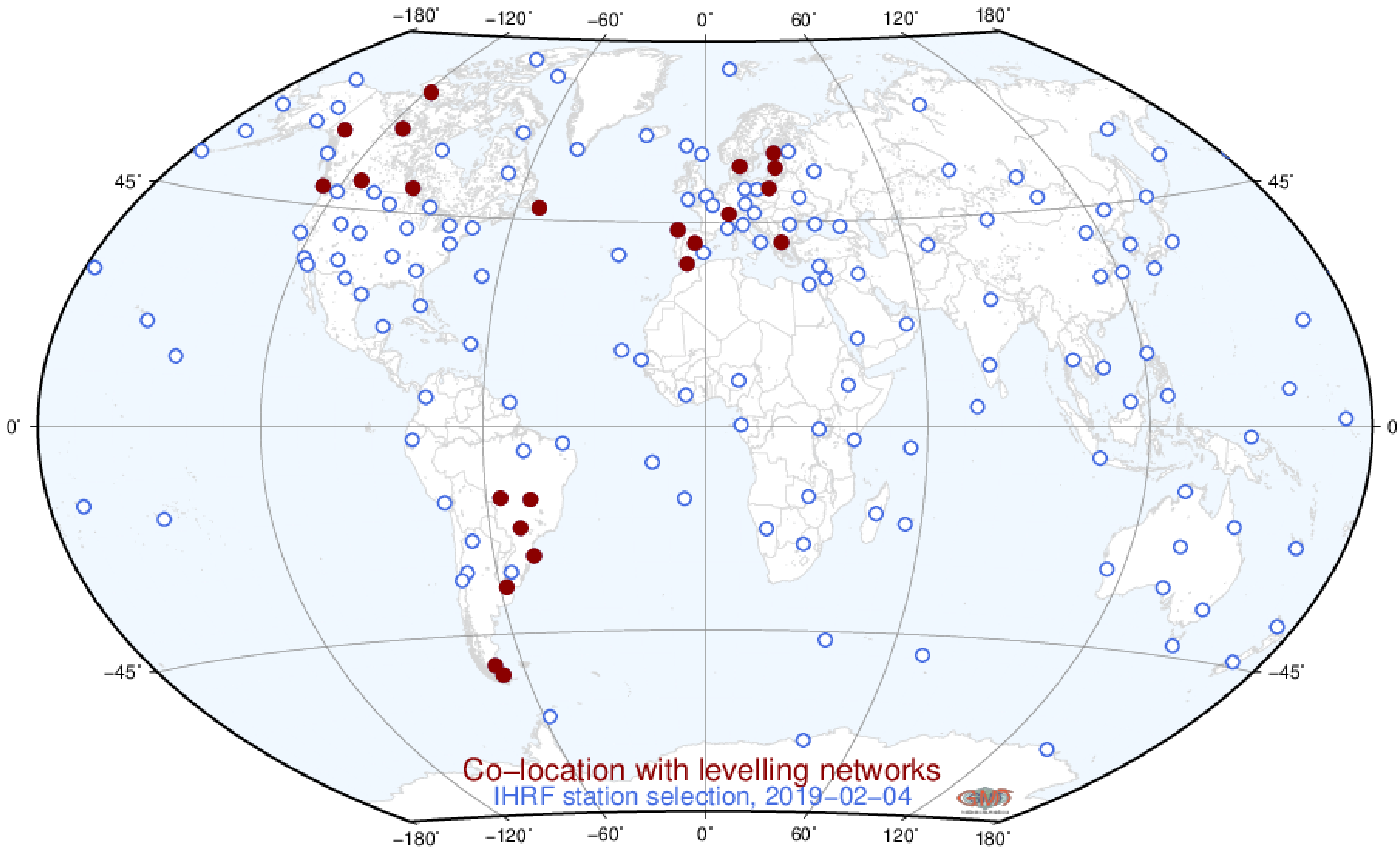
Co-location with absolute gravity (77 sites)



Co-location with tide gauges (26 sites)



Co-location with levelling networks (23 sites)



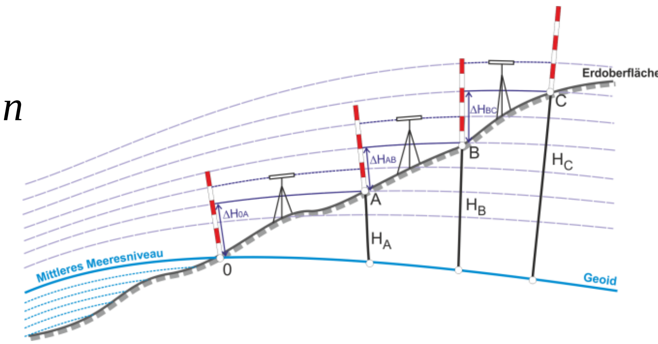
Basic considerations on the IHRS/IHRF coordinates

1) The IHRS/IHRF is the combination of a geometric component given by the coordinate vector **X** in the ITRS/IHRF and a physical component given by the determination of potential values W at **X**.
2) The determination of **X** follows the IERS Conventions. There is not something similar to the IERS Conventions for the determination of W.
3) Current target accuracy for vertical coordinates:
   - Accuracy of the geoid (geometry of any equipotential surface)
     - Static geoid: ± 1 mm, spatial resolution: 10 km.
     - Time-dependent geoid: ± 1 mm, spatial res. 50 km, temporal res. 10 days
   - Accuracy of the ITRF coordinates:
     - Positions: ± 1 mm horizontal, ± 3 mm vertical.
     - Velocities: ± 0.1 mm/a horizontal, ± 0.3 mm/a vertical.
   - Inferred (expected) accuracy for W_P:
     - Positions: $\approx \pm 3\times10^{-2}\ m^2s^{-2}$ (about ± 3 mm).
     - Velocities: $\approx \pm 3\times10^{-3}\ m^2s^{-2}$ (about ± 0.3 mm/a).
4) For the moment, our goal is $\pm1\times10^{-1}\ m^2s^{-2}$ (about 1 cm)
5) The IHRS/IHRF coordinates include the determination of time variations. For the moment, we consider static coordinates only.

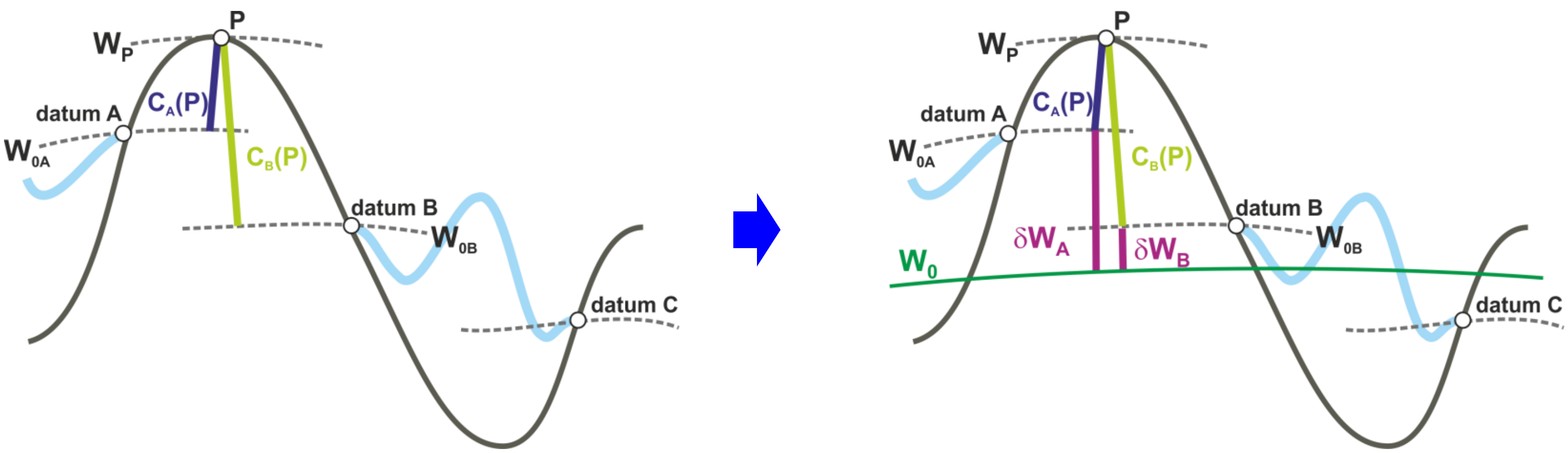
Possibilities for the determination of potential values

1) Geopotential numbers inferred from levelling and gravity reductions:

$$W_P = \left(W_0^{local} + \delta W\right) - C_P; \quad \delta W = W_0^{IHRF} - W_0^{local}\ ;\ C_P = \int_0^P g\, dn$$



- Refer to local vertical datums with unknown potential value $W_{0,local}$ = ?
- To determine W_P, it is necessary to estimate the level difference between the global W_0 and the local $W_{0,local} \rightarrow \delta W = W_0 - W_{0i}$



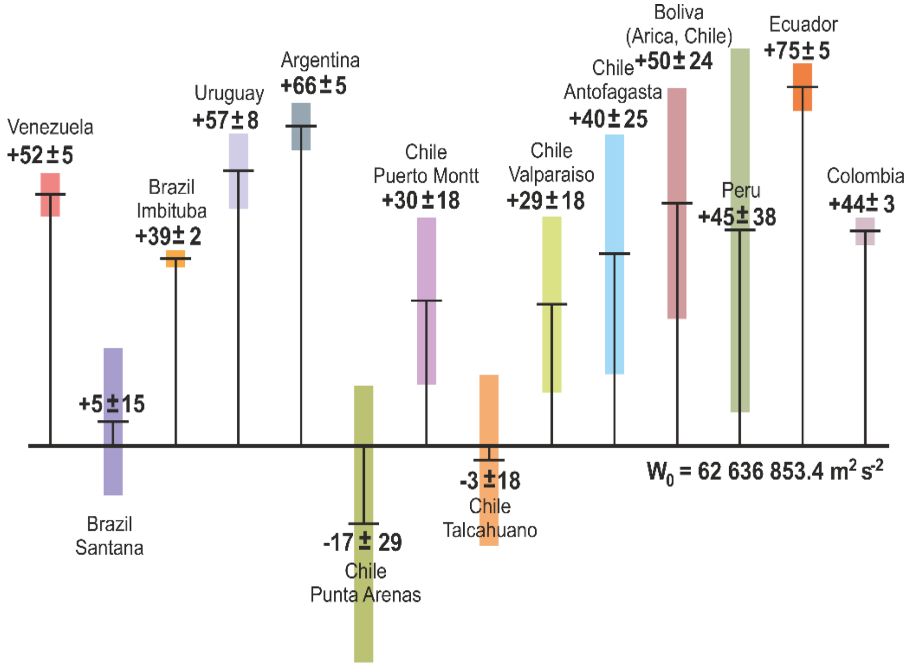
Possibilities for the determination of potential values

1) Geopotential numbers inferred from levelling and gravity reductions:

- Example: δW (in cm) for the South American height systems w.r.t. the IHRS W_0 value.

- Reliability depends on the limitations of the existing height systems, in particular
  - the strong accumulation of systematic errors in levelling, and
  - the impossibility of referring the levelled heights to a specific epoch

- This approach is useful for the transformation of the existing height systems to the IHRS, but it may be unsuitable for the precise realisation of the IHRS.



Possibilities for the determination of potential values

IUGG

IAG AIG International Association of Geodesy

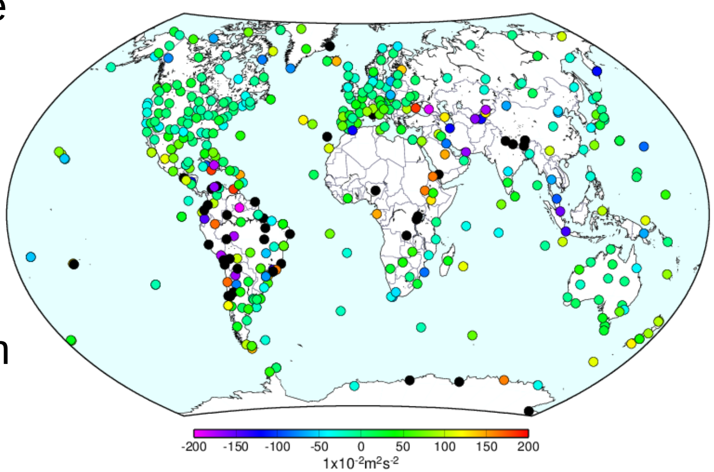
TUM

2) Global Gravity Models of high degree (GGM-HD) like the EGM2008 model (Pavlis et al., 2012, 2013) or the EIGEN-C series (e.g., Förste et al., 2012; 2014)

$$W(X,Y,Z)=\frac{GM}{r}\left[1+\sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}\left[C_{nm}\cos m\lambda+S_{nm}\sin m\lambda\right]P_{nm(\cos\theta)}\right]+\frac{1}{2}\omega^2r^2\cos(90°-\theta)$$

- Expected accuracy (Rummel et al., 2014)
  - well surveyed regions: ±0.4 m^2s^{-2} to ±0.6 m^2s^{-2} (equivalent to ±4 cm to ±6 cm)
  - sparsely surveyed regions: from ±2 m^2s^{-2} ... ±4 m^2s^{-2} (±20 cm to ±40 cm) to ±10 m^2s^{-2} (±1 m)

- Differences between the W_P values derived from EGM2008 (Pavlis et al. 2008) and EIGEN6C4 (Förste et al. 2014), both at n=2190
  - Differences larger than ±200 x 10^{-2} m^2s^{-2} (≈ ± 2 m)
  - Desired accuracy for W_P: ±0.03 m^2s^{-2} (≈ ± 3 mm)



- This approach represents the "ideal way" to estimate potential values and hopefully, we will get a better accuracy in the next future. Ongoing studies with high expectation of improvement:
  - Combination of GGM with gravity effects of global topography
  - EGM2020
  - However, terrestrial gravity data is further required!

Possibilities for the determination of potential values

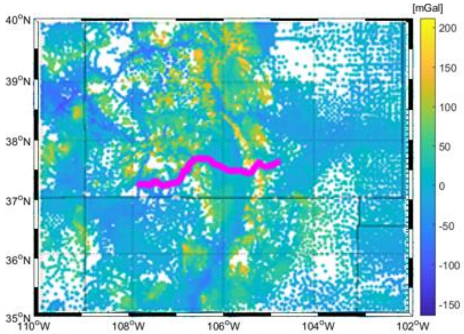
3) Disturbing potential

$$W_P = U_P + T_P \quad ; \quad T_P = T_{P,satellite-only} + T_{P,residual} + T_{P,terrain}$$

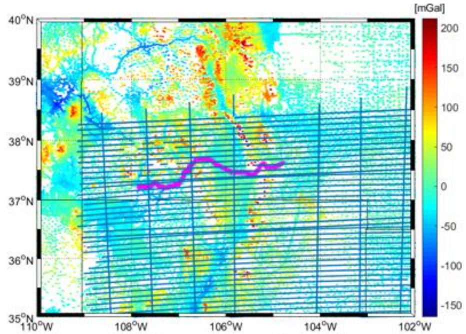
- GGM based on SLR, GRACE and GOCE are very precise
  (±1 ... ±2 cm @ 100 km)
- Mean omission error globally: ≈ ±45 cm
- Goal is to reduce these ±45 cm to ±1 cm (only possible using terrestrial gravity data and considering topographic effects)
- The potential values realising the IHRS coordinates must be determined at the reference stations; i.e., at the Earth's surface and not at the geoid
- The determination of T_P demands a series of approximations, which influence the results; i.e., different methodologies produce different potential values

Comparison of computation methods

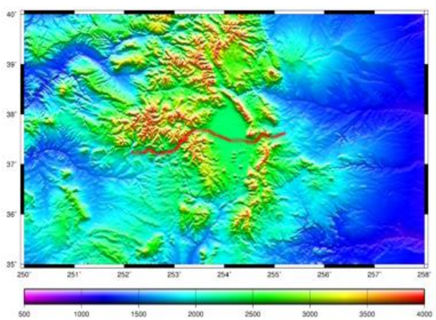
Colorado experiment: to compute geoid, quasi-geoid and potential values using exactly the same input data, a set of basic standards, and the own methodologies (software) of colleagues involved in the gravity field modelling.

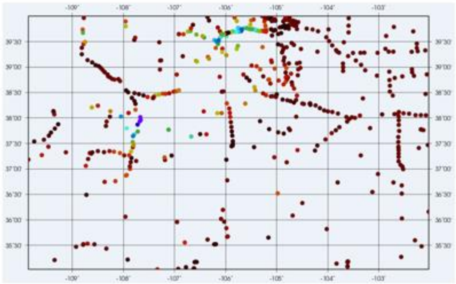


Surface gravity data (59,303 points)



Airborne gravity data
(41 lines E-W, 7 lines N-S)


Terrain model: SMRT V4.1


NGS *historical* GPS/levelling (510 points)

- Initiated in July 2017
- Data provided by US NGS
- Standards prepared by L Sánchez, J Ågren, J Huang, YM Wang, R Forsberg
- Three computations (two iterations) finished in June 2019
- Fifteen (final) contributing solutions
- Special Issue of the Journal of Geodesy with computation methods and comparison of geoid and quasi-geoid models (in preparation).

Colorado experiment: contributing solutions

Faculty of Engineering, Minia University, Egypt

İstanbul Teknik Üniversitesi, Istambul, Turkey

Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Thessaloniki, Greece

National Geodetic Survey, USA

Natural Resources Canada, Canada

Lantmäteriet, Swedish mapping, cadastral and land registration authority, Sweden

School of Earth and Planetary Sciences and The Institute for Geoscience Research, Curtin University, Australia

Escola Politécnica, Universidade de São Paulo; Centro de Estudos de Geodesia, Brazil

Deutsches Geodätisches Forschungsinstitut, Technische Universität München, Germany

Ingenieurinstitut für Astronomische und Physikalische Geodäsie, Technische Universität München, Germany

Chinese Academy of Surveying and Mapping, China

Politecnico de Milano, Italy

Faculty of Geodesy, University of Zagreb, Croatia - Research Institute of Geodesy, Topography and Cartography, Czech Republic

National Space Institute, Technical University of Denmark, Denmark

Geography and Crustal Dynamics Research Center, Geospatial Information Authority of Japan, Japan

Colorado experiment: summary of approaches and models

- Least squares modification of Stokes’ formula with additive corrections (2)
- Least squares modification of Stokes’ formula with additive corrections and biased Stokes’ kernel modification
- Stokes’ formula with Wong-Gore modification and 1D-FFT (2)
- Spherical radial basis functions
- Least squares collocation
- Fast collocation based on gravity gridded data
- Degree weighted Stokes’ integral
- Modified degree-banded Stokes‘ kernel (2)
- Spherical FFT with modified Wong-Core Stokes‘ kernel
- UNB Stokes-Helmert scheme
- UNB Stokes-Helmert scheme with hybrid-Meissl-Molodensky modified spheroidal Stokes‘ kernel
- NGS Molodensky approach, Spherical Harmonics Analysis (SHA)
- GGMs: GOCO05s, XGM2016, XGM2018, xGEOID17B, EIGEN-6C4, EGM2008
- Topographic effects based on SRTM V4.1, EARTH2014, COLH19M05, ERTM2160
- 12 solutions based on height anomalies, 3 solutions based on geoid undulations

Colorado experiment: comparison of potential values

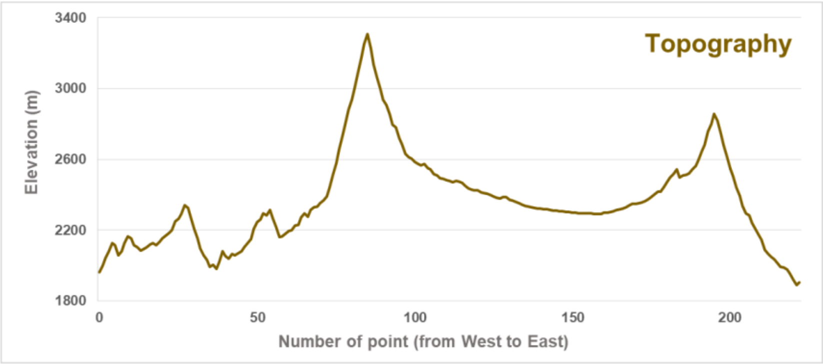
1) The comparison is carried out at 223 GSVS17 marks (Geoid Slope Validation Survey 2017) selected by NGS

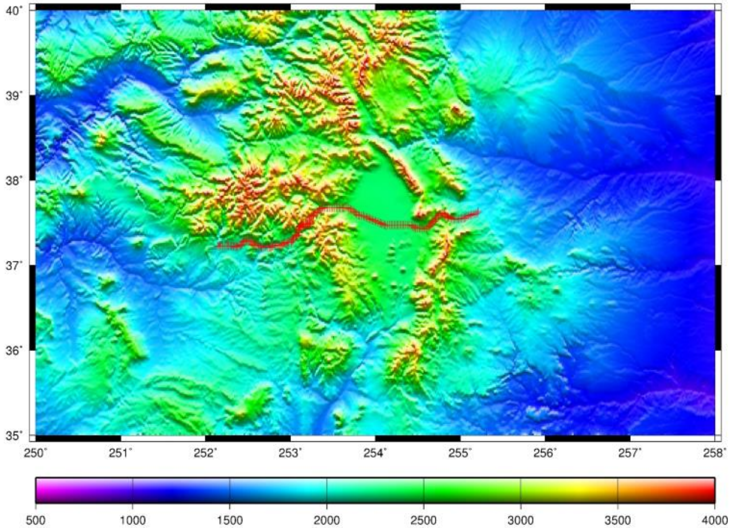
2) Participants in the experiment got φ, λ, h; levelling is not available (yet)

3) The potential values provided by the different solutions are converted to geopotential numbers with respect to the IHRS W_0 value

$$C(P) = W_0 - W(P) \quad ; \quad W_0 = 62\,636\,853.4 \text{ m}^2\text{s}^{-2}$$

4) and further transformed to normal heights (to see the differences in meters): $H^*(P) = C(P)/\gamma(P)$

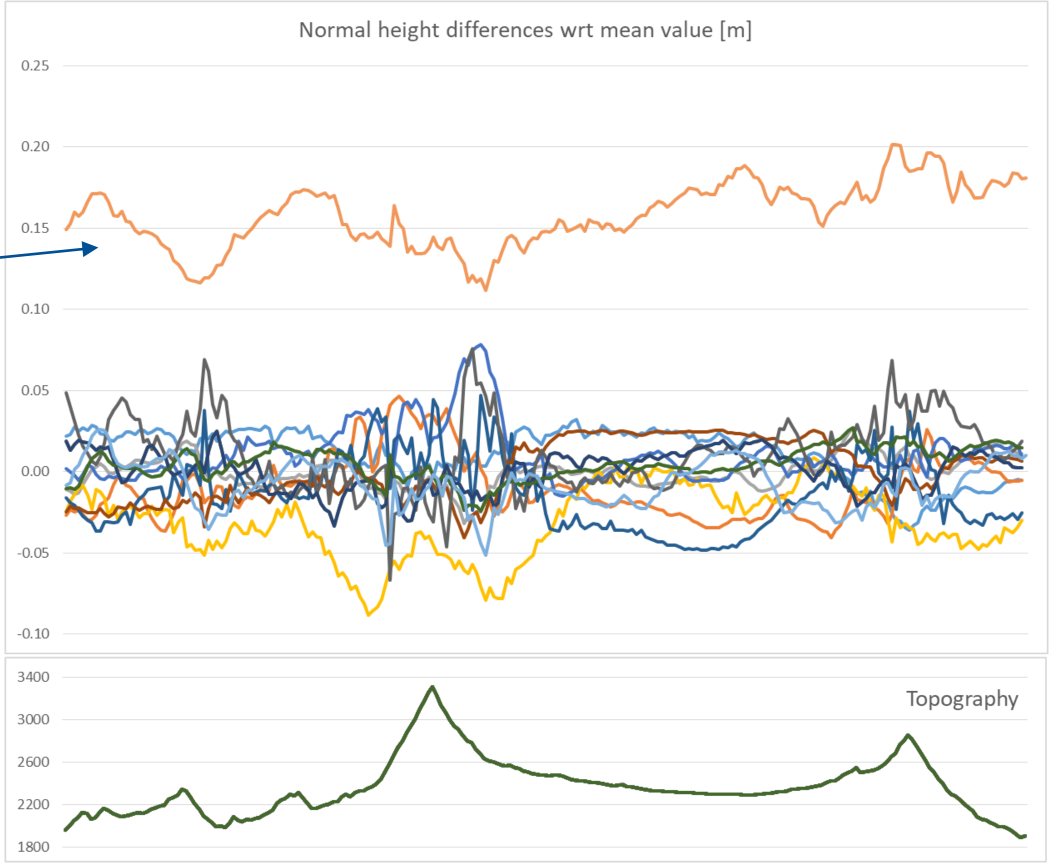




Colorado experiment: comparison of potential values

Normal height difference [cm]
(individual contribution – mean)

Outlier 1
Mean : 15.7 ± 1.9 cm
Range: 8.9 cm (11.2 ... 20.0 cm)



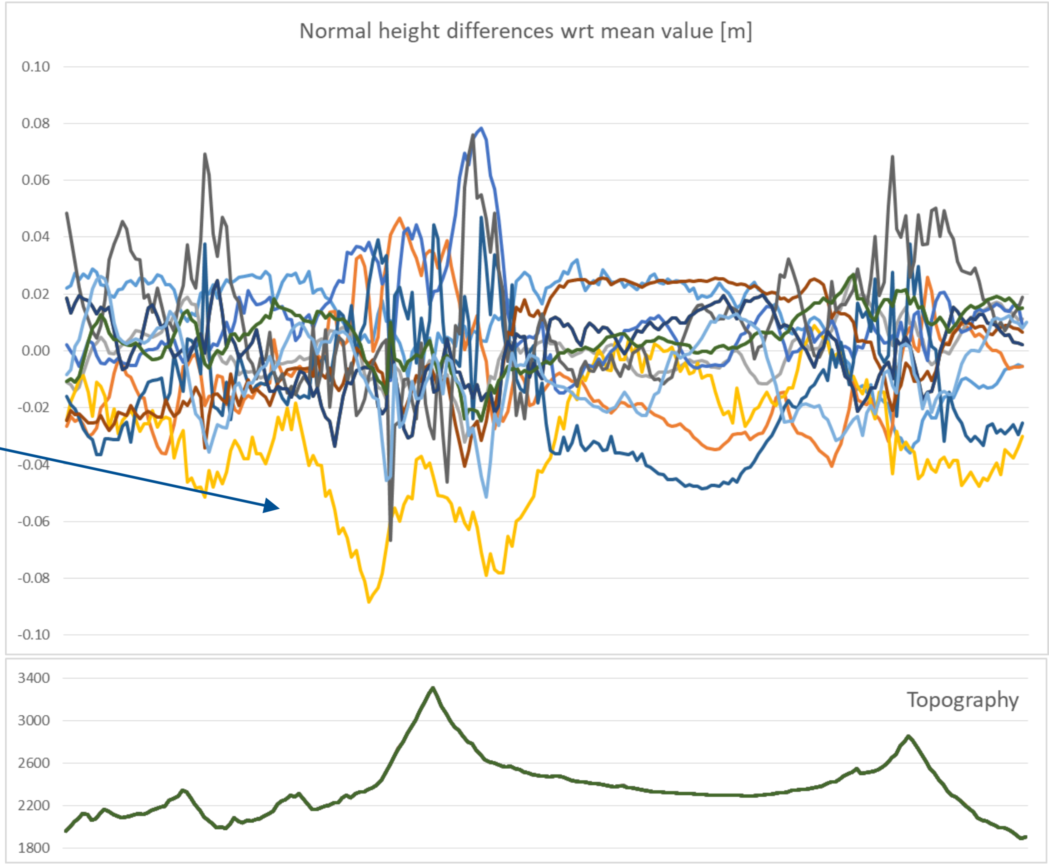
Zero-degree term: 17.85 cm

$$\zeta_0 = \frac{\left(GM_{GGM} - GM_{GRS80}\right)}{r_P \cdot \gamma_Q} - \frac{\Delta W_0}{\gamma_Q}$$

Colorado experiment: comparison of potential values

Normal height difference [cm]
(individual contribution – mean)

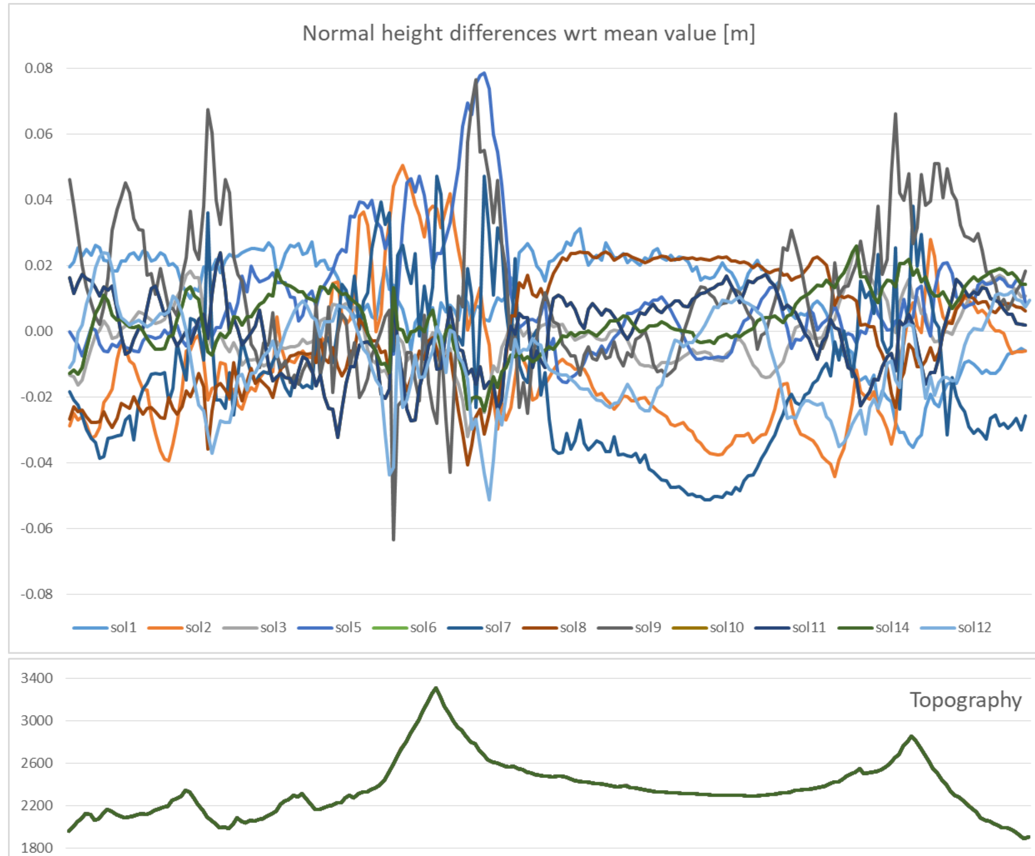
Outlier 2
Mean : -3.2 ± 2.1 cm
Range: 9.3 cm (-8.7 ... 0.6 cm)



Colorado experiment: comparison of potential values

Normal height difference [cm]
(individual contribution – mean)

	Mean	±	STD	Range					
sol1	1.0	±	1.6	6.6	(	-3.5	...	3.1	)
sol2	-1.0	±	2.1	9.5	(	-4.4	...	5.1	)
sol3	-0.1	±	1.0	5.8	(	-3.2	...	2.6	)
sol5	1.0	±	1.8	9.4	(	-1.6	...	7.9	)
sol6	0.4	±	1.0	5.3	(	-2.7	...	2.6	)
sol7	-1.4	±	2.3	9.9	(	-5.1	...	4.7	)
sol8	0.0	±	1.8	6.5	(	-4.1	...	2.4	)
sol9	1.1	±	2.2	14.0	(	-6.3	...	7.7	)
sol10	0.0	±	1.2	7.5	(	-3.2	...	4.3	)
sol11	0.0	±	1.1	5.6	(	-3.2	...	2.4	)
sol12	-0.9	±	1.4	7.5	(	-5.1	...	2.4	)
sol14	0.4	±	1.0	5.0	(	-2.4	...	2.6	)

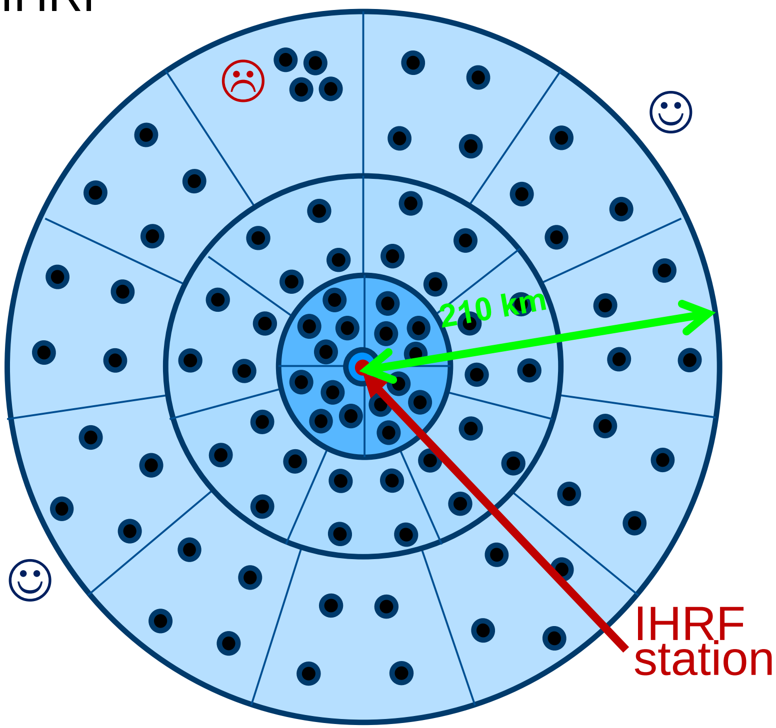


Learnings from the Colorado experiment

1) Validation of gravity field (geoid) modelling additional to GNSS/levelling
2) Twelve(!) solutions agree within 1 cm to 2 cm in terms of standard deviation with respect to the mean value
3) We are waiting for the levelling results along the test profile to make comparisons with independent data
4) Discrepancies between the different solutions are highly correlated with the topography
   - Handling of terrain gravity effects (model and strategy)
5) Difficulties reported by the colleagues contributing to the experiment
   - Processing of the airborne gravity data
   - Handling of the zero-degree term
6) A major confusion is the reference ellipsoid: which should be used GRS80 or WGS84?
   - Are we needing a new reference ellipsoid?

Learnings of the Colorado experiment

- The GGM should be based at least on the combination of SLR, GRACE and GOCE data (n ≥ 200)
- To get an accuracy of about ±1 cm in the (quasi-)geoid, observed gravity values are required with a mean spatial resolution of about 4 km
- The availability of these data is a main criterion to select reference stations for the IHRF



Template according to the gravity effect on the geoid ($\Delta g = 1 \cdot 10^{-6}$ $ms^{-2} \rightarrow 1$ mm)

Distance	Compartments	# of points flat/mountain
10 km	1	**4/8**
50 km	4	**20/30**
110 km	7	**30/45**
210 km	11	**50/75**
Sum	23	**104/158**

Outlook

1) To compute a first static solution for the IHRF to evaluate the achievable accuracy under the present conditions (data availability, computation methods, etc.) and to identify key actions to improve the determination of the IHRS/IHRF coordinates.
2) To investigate the determination of potential changes with time $\dot{W}$.
3) To extend the realisation of the IHRS to marine areas.
4) To explore the possibilities to establish an 'IHRS/IHRF element' within the International Gravity Field Service (IGFS) to ensure the maintenance and availability of the IHRF:
   - Regular updates of the IHRFyyyy to take account for:
   - new stations;
   - coordinate changes with time $\dot{\mathbf{X}}$, $\dot{W}$;
   - improvements in the estimation of $\mathbf{X}$ and W (more observations, better standards, better models, better computation algorithms, etc.)

Participation of Latin America in the implementation of the IHRS/IHRF

1) Establishment of IHRS stations
   - To select some (1 to 5) continuously operating SIRGAS reference stations in each country (well distributed and materialized by a monument on the ground; stations on the top of buildings are not welcome).
   - To survey gravity data around the selected SIRGAS reference stations (about 150 gravity points well distributed around each station up to a distance of about 200 km).
   - Coordinates of gravity points determined with GNSS positioning ($\pm$2 cm).
   - It is desirable that the gravity surveys refer to absolute gravity stations.

2) Integration of the existing Latin American height systems into the IHRS/IHRF
   - First order levelling (with gravity data) of SIRGAS reference stations (optimal if IHRF stations are levelled).
   - Reference tide gauges connected to SIRGAS.
   - Combination of ellipsoidal heights, levelling-based physical heights, tide gauge registrations, satellite altimetry observations and height-resolution gravity field modelling.

3) Latin American countries should take advantage of the SIRGAS-WG3 activities:
   - Capacity building and software for the processing of gravity data
   - Capacity building and software for the adjustment of levelling networks and computation of geopotential numbers
   - Until now: Rio (2012), La Paz (2014), Curitiba (2015), Quito (2016), San José (2017), Aguascalientes (2018)
   - Once the levelling networks are properly adjusted, a workshop about the integration of the existing height systems into the IHRS/IHRF can be planned.

Acknowledgment

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- GGOS JWG: Strategy for the realisation of the IHRS (chair: L Sánchez)
- IAG JWG 2.2.2: The 1 cm geoid experiment (chair: YM Wang)
- IAG SC 2.2: Methodology for geoid and physical height systems (chair: J Ågren)
- ICCT JSG 0.15: Regional geoid/quasi-geoid modelling - Theoretical framework for the sub-centimetre accuracy (chair: J Huang)
- IAG JWG 2.1.1: Establishment of a global absolute gravity reference system (chair: H Wziontek)
- IAG regional sub-commissions for reference frames and geoid modelling
- IAG Commission 2 – Gravity Field (chair R Pail)
- International Gravity Field Service – IGFS (chair R Barzaghi)
- GGOS Bureaus of Networks and Observations – GGOS-BNO (chair: M Pearlman) and Products and Standards – GGOS-BPS (chair: D Angermann)